α_s and the τ hadronic width

Based on the recent article:

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Introduction

- Investigations of hadronic τ decays already contributed tremendously for fundamental QCD parameters like α_s, the strange mass and non-perturbative condensates.

- In particular: (Davier et al. 2008)

$$\alpha_s(M_\tau) = 0.344 \pm 0.005_{\rm exp} \pm 0.007_{\rm th} \,,$$

leading to

$$\alpha_s(M_Z) = 0.1212 \pm 0.0011 \,.$$

- This should be compared to the recent average: (Bethke 2007)

$$\alpha_s(M_Z) = 0.1185 \pm 0.0010 \,,$$

being significantly lower.

Hadronic τ decay rate

Consider the physical quantity R_τ: (Braaten, Narison, Pich 1992)

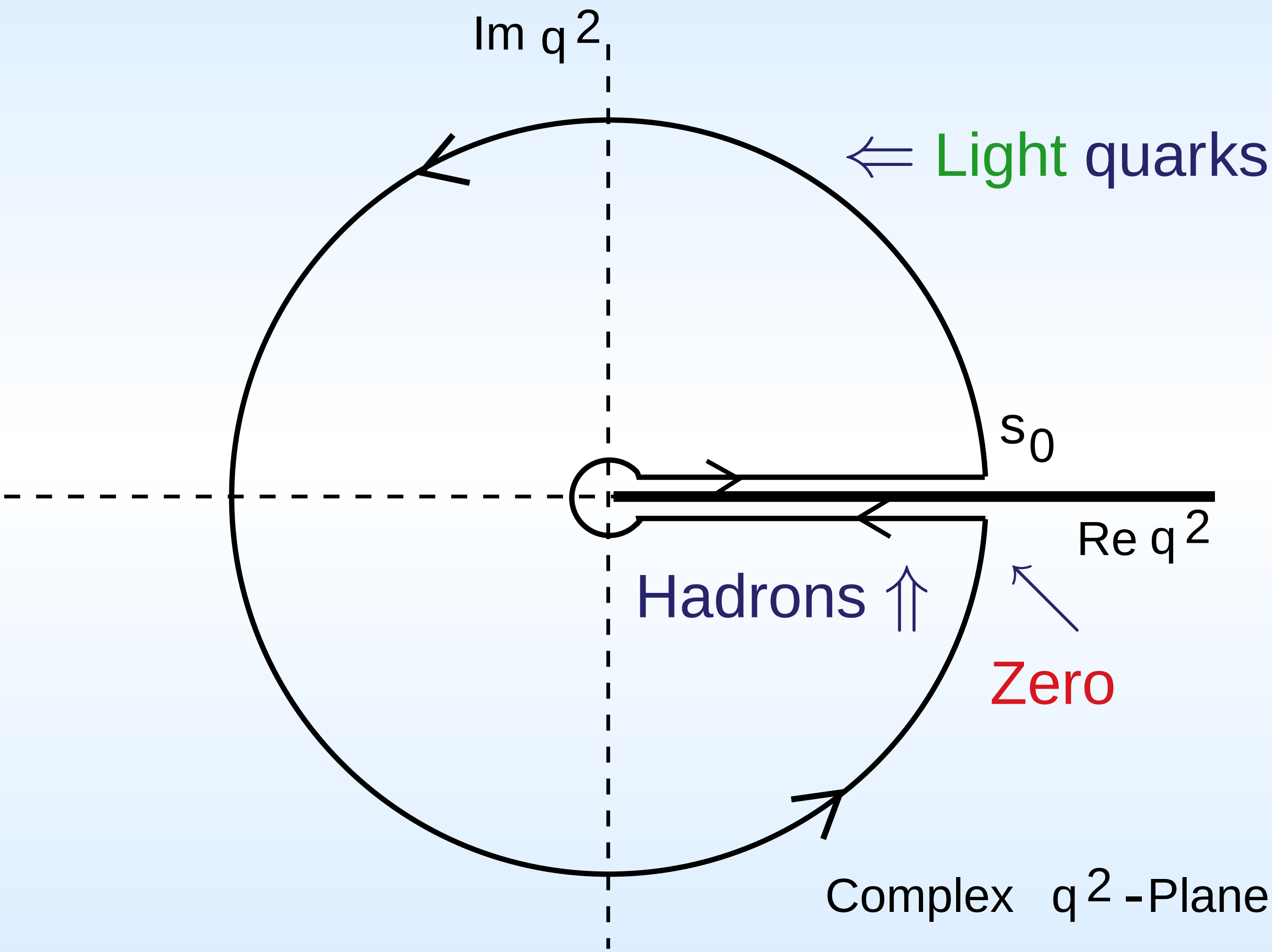
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar\nu_e\nu_\tau(\gamma))} = 3.640 \pm 0.010\,.$$

Theoretically, R_τ can be expressed as:

$$R_\tau = N_c\, S_{\rm EW} \Big\{ (|V_{ud}|^2 + |V_{us}|^2)\big[1 + \delta^{(0)}\big] + \sum_{D\geq 2}\Big[|V_{ud}|^2\delta_{ud}^{(D)} + |V_{us}|^2\delta_{us}^{(D)}\Big]\Big\}\,.$$

$\delta_{ud}^{(D)}$ and $\delta_{us}^{(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle\bar q q\rangle$.

Hadronic τ decay rate



Adler function

The perturbative part $\delta^{(0)}$ is related to the Adler function $D(s)$:

$$D(s) \equiv -s\,\frac{\mathrm{d}}{\mathrm{d}s}\,\Pi_V(s) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\,c_{n,k}\,\ln^{k-1}\Big(\frac{-s}{\mu^2}\Big)$$

where $a_\mu \equiv \alpha_s(\mu)/\pi$.

Resumming the Log's with the scale choice $\mu^2 = -s \equiv Q^2$:

$$D(Q^2) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} c_{n,1}\,a^n(Q^2)$$

As a consequence, only the coefficients $c_{n,1}$ are independent:

$$c_{0,1} = c_{11} = 1\,, \quad c_{2,1} = 1.640\,, \quad c_{3,1} = 6.371\,,$$

$$c_{4,1} = 49.076\,!!$$

(Baikov, Chetyrkin, Kühn 2008)

Fixed order perturbation theory amounts to choose $\mu^2 = M_\tau^2$:

$$\delta^{(0)}_{\rm FO} = \sum_{n=0}^{\infty} a^n(M_\tau^2) \sum_{k=1}^{n+1} k\, c_{n,k} J_{k-1} = \sum_{n=0}^{\infty} [c_{n,1}+g_n] a^n(M_\tau^2)$$

A given perturbative order n depends on all coefficients $c_{m,1}$ with $m \leq n$, and on the coefficients of the QCD β-function.

Contour improved perturbation theory employs $\mu^2 = -M_\tau^2 x$:

(Pivovarov; Le Diberder, Pich 1992)

$$\delta^{(0)}_{\rm CI} = \sum_{n=0}^{\infty} c_{n,1} J_n^a(M_\tau^2) \qquad \text{with}$$

$$J_n^a(M_\tau^2) = \frac{1}{2\pi i} \oint\limits_{|x|=1} \frac{dx}{x} (1-x)^3 (1+x) a^n(-M_\tau^2 x)$$

Employing $\alpha_s(M_\tau) = 0.34$, the numerical analysis results in:

$$\begin{array}{llllll} & a^1 & a^2 & a^3 & a^4 & a^5 \\ \delta^{(0)}_{\rm FO} = & 0.108 + & 0.061 + & 0.033 + & 0.017 & (+0.009) = 0.220\,(0.229) \\ \delta^{(0)}_{\rm CI} = & 0.148 + & 0.030 + & 0.012 + & 0.009 & (+0.004) = 0.198\,(0.202) \end{array}$$

Contour improved PT appears to be better convergent.

The difference between both approaches amounts to 0.022 !

From the uniform convergence of $\delta^{(0)}_{\rm FO}$, and the assumption that the series is not yet asymptotic, one may also infer

$$c_{5,1} \approx 283\,,$$

leading to a difference of $\delta^{(0)}_{\rm FO} - \delta^{(0)}_{\rm CI} = 0.027$.

Borel transform

To further investigate the difference between CI and FOPT, we propose to model the Borel-transformed Adler function.

$$4\pi^2 D(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(s)^{n+1} \,,$$

where $r_n = c_{n+1,1}/\pi^{n+1}$. The Borel-transform reads:

$$\widehat{D}(\alpha_s) = \int_0^\infty \mathrm{d}t \, \mathrm{e}^{-t/\alpha_s} B[\widehat{D}](t) \,; \quad B[\widehat{D}](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!} \,.$$

Generally, the Borel-transform $B[\widehat{D}]$ developes poles and cuts at integer values p of the variable $u \equiv \beta_0 t$. (Except at $u=1$.)

The poles at negative p are called UV renormalon poles and the ones at positive p IR renormalons.

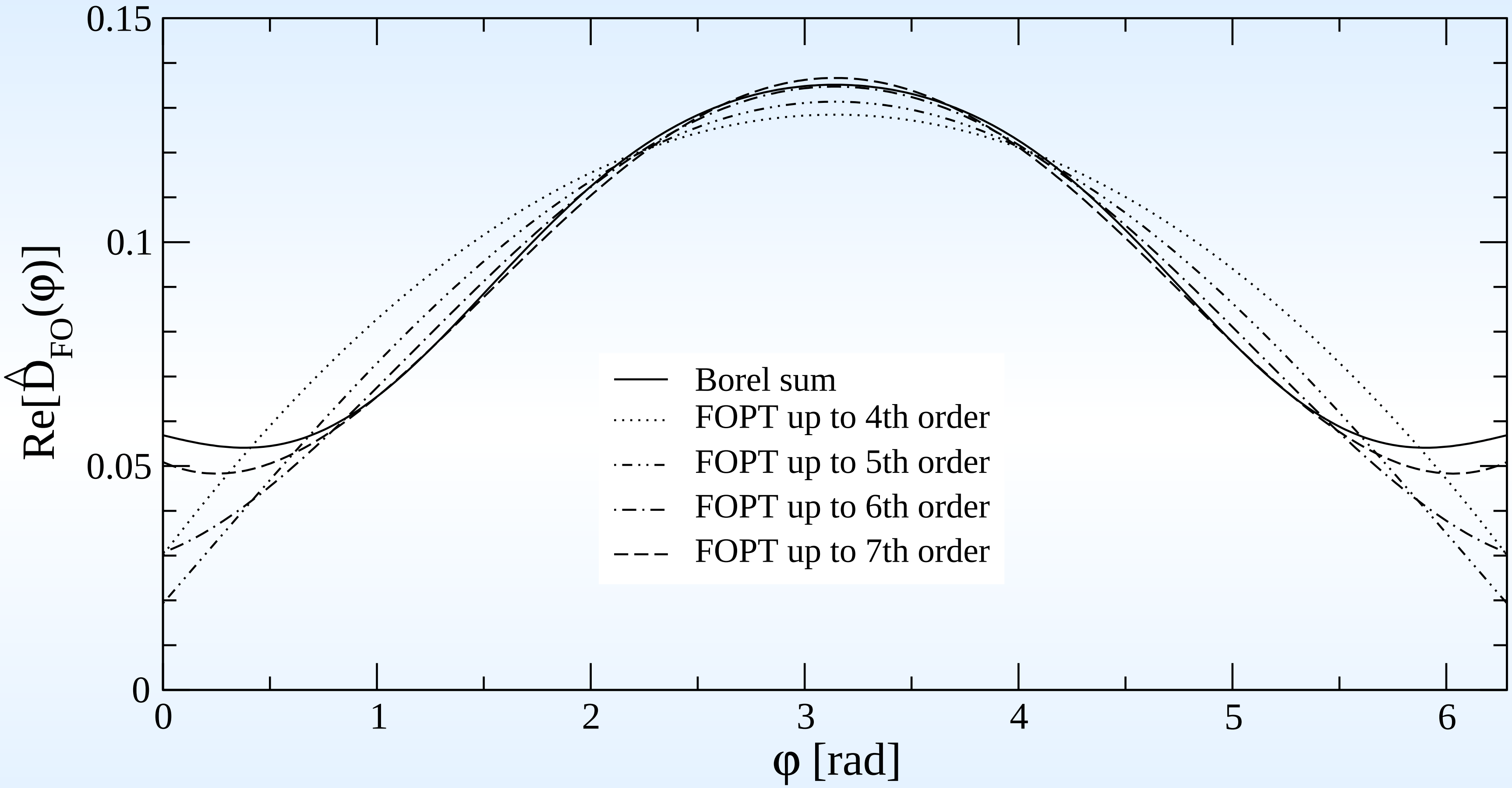
$$B[\widehat{D}](u) \; = \; B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_2^{\rm IR}](u) + B[\widehat{D}_1^{\rm IR}](u)$$
$$+\, d_0^{\rm PO} + d_1^{\rm PO}\, u\,,$$

where

$$B[\widehat{D}_p](u) = \frac{d_p}{(p\pm u)^{1+\gamma}}\,\big[\,1 + b_1(p\pm u) + b_2(p\pm u)^2\,\big]\,.$$

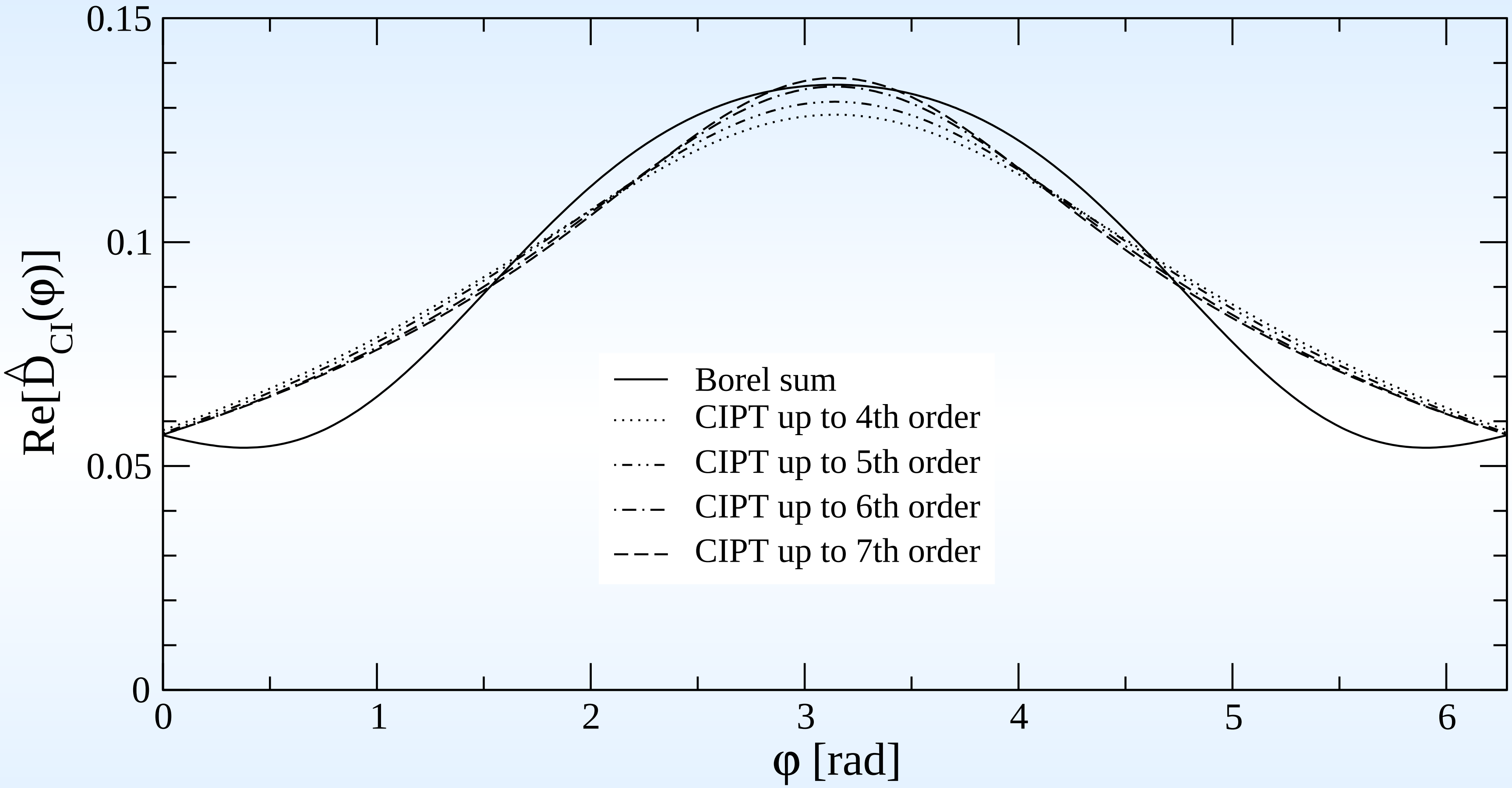
- ☞ Our model incoporates the leading UV pole ($u=-1$), as well as the two leading IR renormalons ($u=2,3$).
- ☞ It should reproduce the exactly known $c_{n,1}$, $n \leq 4$.
- ☞ For both UV and IR, the residues d_p are free while γ, $b_{1,2}$ depend on anomalous dimensions and β-coefficients.

Adler function



$$c_{5,1} = 283\,, \quad \alpha_s(M_\tau) = 0.3156\,.$$

Adler function



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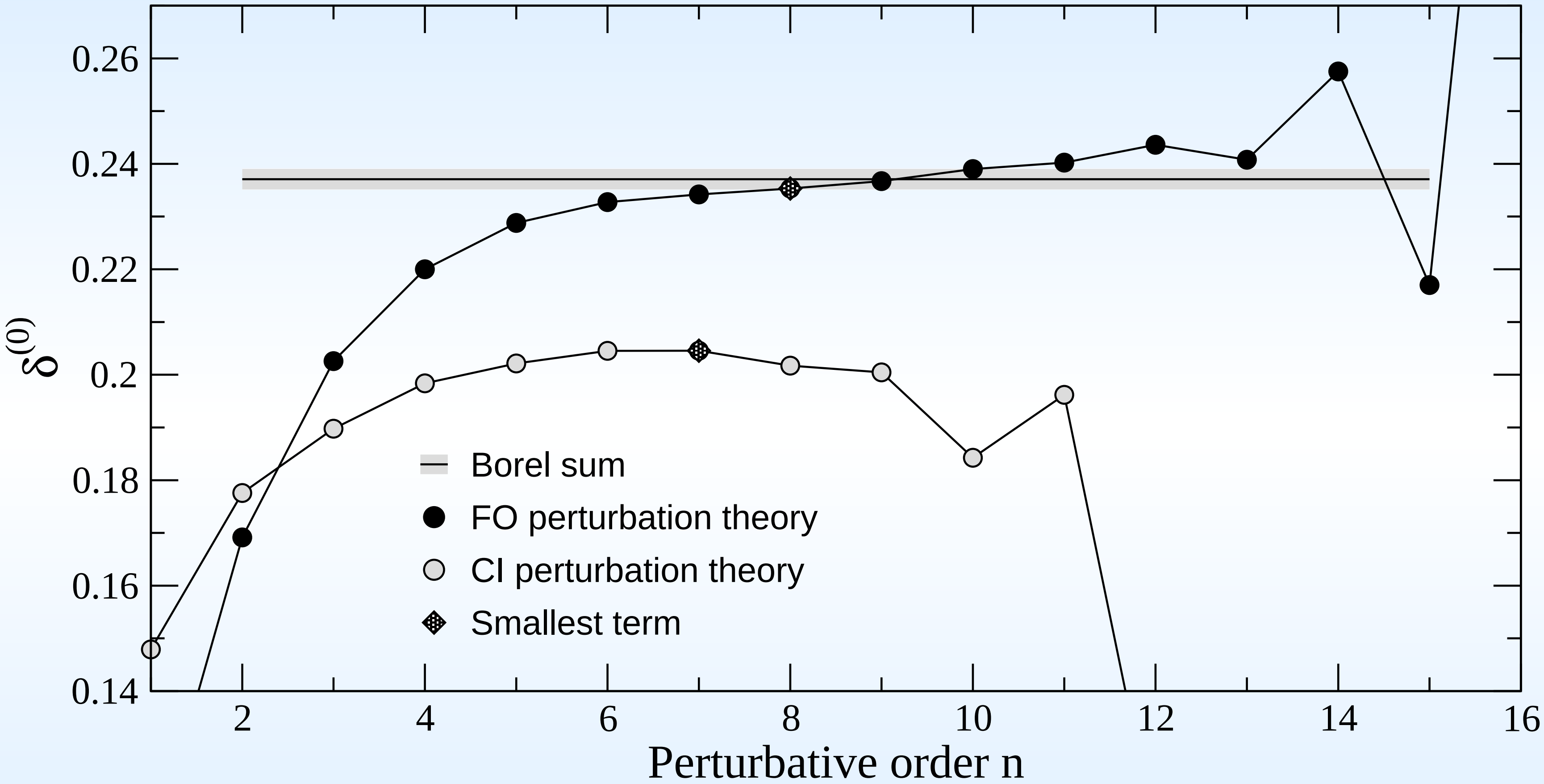
Borel model



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Employing the hadronic decay rate into light quarks

$$R_{\tau,V+A} = N_c\,|V_{ud}|^2\,S_{\rm EW}\Big[\,1+\delta^{(0)}+\delta^{\rm NP}_{V+A}\,\Big]$$

with $\delta^{\rm NP}_{V+A} = (-\,7.1 \pm 3.1)\cdot 10^{-3}$, one finds

$$\delta^{(0)} = \frac{R_{\tau,V+A}}{3|V_{ud}|^2 S_{\rm EW}} - 1 - \delta^{\rm NP}_{V+A} = 0.2042(38)(33)$$

The first uncertainty is due to $R_{\tau,V+A}$, while the remaining error is dominated by $\delta^{\rm NP}_{V+A}$.

Scanning over plausible models and adjusting α_s such as to reproduce $\delta^{(0)}$, one finally obtains

$$\alpha_s(M_\tau) = 0.3156(30)(51) \;\Rightarrow\; \alpha_s(M_Z) = 0.1180(8)$$

Conclusions

FOPT povides the closer approach to the “true” (Borel resummed) value for the perturbative correction $\delta^{(0)}$.

CIPT fails to yield the correct result, as it misses the large cancellations between the coefficients $c_{n,1}$ and the g_n.

This behaviour is particular for R_τ, due to the strong suppression of the leading IR renormalon at $u = 2$.

Our central result is $\alpha_s(M_\tau) = 0.3156(59)$, leading to $\alpha_s(M_Z) = 0.1180(8)$ after evolution to M_Z, significantly lower than previous α_s values from R_τ.

Work on m^2 and scalar contributions in process with F. Schwab.

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Thank You for Your attention !