α_s FROM HADRONIC τ DECAYS

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Tau 2008, Novosibirsk

OUTLINE

- *A few relevant background points*
- *Problems (possible and actual) with the conventional spectral weight analysis*
- *An alternate approach*
- *Results etc*

A FEW BACKGROUND POINTS

- Notation:
  - $R_{V/A;ud} \equiv \frac{\Gamma[\tau^- \to \nu_\tau \, \text{hadrons}_{V/A;ud}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$
  - kinematic transverse and longitudinal weights: $w_{(00)}(y) \equiv (1-y)^2(1+2y)$, $w^L_{(00)}(y) \equiv -2y(1-y)^2$
  - $a(Q^2) \equiv \alpha_s(Q^2)/\pi$
  - $\Pi^{(J)}_{V/A;ij}(s)$, $\rho^{(J)}_{V/A;ij}(s)$: flavor ij, $J = 0, 1$ scalar part and corresponding spectral function of V/A current-current correlator

- In the SM, with $y_\tau = s/m_\tau^2$,

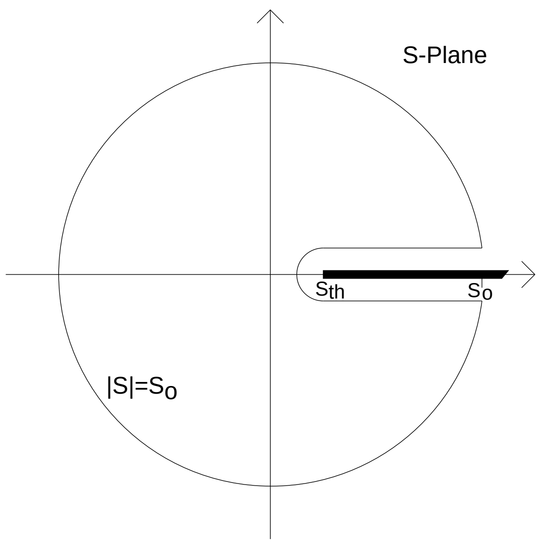
$$\frac{dR_{V/A;ud}}{ds} = \frac{12\pi^2 |V_{ud}|^2 S_{EW}}{m_\tau^2} \left[w_{(00)}(y_\tau)\rho^{(0+1)}_{V/A;ud}(s) + w^L_{(00)}(y_\tau)\rho^{(0)}_{V/A;ud}(s) \right]$$

  - $\rho^{(0)}_{V;ud}(s)$: $O\left[(m_d - m_u)^2\right]$, hence negligible
  - $\rho^{(0)}_{A;ud}(s) = 2f_\pi^2 \delta(s - m_\pi^2)$ up to negligible $O\left[(m_d + m_u)^2\right]$ corrections
  - $\Rightarrow \rho^{(0+1)}_{V/A;ud}(s)$ from measured experimental $dR_{V/A;ud}/ds$ distribution

- FESRs: $w(s)$ analytic, $\Pi(s)$ no kinematic singularities

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

(data on LHS, OPE on RHS)



- DATA: here, $\rho(s) = \rho^{(0+1)}_{V,A,V+A;ud}(s)$ from $dR_{V,A,V+A;ud}/ds$

- OPE:

  - $w(s_0) = 0$ ("pinching"), $s_0 \gtrsim 2$ GeV^2 to avoid OPE breakdown

  - $D = 0$: fixed by α_s (expansion known to 5 loops)

  - $D = 2$: $O\left[(m_d \mp m_u)^2, \alpha_s^2 m_s^2\right]$, hence tiny

  - $D = 4$: fixed by $\langle aG^2\rangle$, $\langle m_\ell \bar{\ell}\ell\rangle$, $\langle m_s \bar{s}s\rangle$

  - $D = 6, 8, \cdots$ not known phenomenologically

- FESR OPE features

  - V, A, V+A correlator OPE *strongly* $D=0$ dominated for $s_0 \gtrsim 2$ GeV^2 $\Rightarrow$ good sensitivity to α_s

  - good convergence of integrated $D=0$ series

  - for $w(s)\rightarrow w(y)$, $y=s/s_0$, integrated $D=2k+2$ OPE contribution scales as $1/s_0^k$

  - integrated $D=2k+2\geq 2$ contribution for degree N $w(y)=\sum_{m=0}^{N} b_m y^m \Leftrightarrow b_k\neq 0$ (up to $O[\alpha_s^2(s_0)]$ corrections) $\Rightarrow$ contributions up to $D_{max}=2N+2$

  - With $\left[\Pi(Q^2)\right]^D_{OPE}=\frac{C_D}{Q^D}$,

$$\left[\frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(y)\,\Pi(s)\right]^{D>4}_{OPE} = \sum_{m=2}(-1)^m \frac{b_m C_{2m+2}}{s_0^m}$$

- despite $D = 0$ dominance, high degree of control of smaller higher D non-perturbative contributions needed for precision α_s determination

  - $\alpha_s(M_Z)$ to $\sim 1\%$ $\leftrightarrow$ $\alpha_s(m_\tau)$ to $\sim 3\%$

  - integrated $D = 0$ OPE contributions

    $$\sim C_w \left[1 + a(s_0) + \cdots\right]$$

    with C_w, $O(a^2(s_0))$ terms $w(y)$-dependent

  - $a(m_\tau) \sim 0.10$

  $\Rightarrow$ need higher D contributions under control to $\sim 0.3\%$ of the full OPE ($\sim 0.5\%$ if take higher order $D = 0$ terms into account)

THE (km) SPECTRAL WEIGHT ANALYSIS

- $w_{(00)}(y) = 1 - 3y^2 + 2y^3 \Rightarrow$ "unsuppressed" OPE contributions in $w_{(00)}$ FESR up to $D = 6, 8$

- $\Gamma[\tau \rightarrow hadrons_{ud}\, \nu_\tau]$ from inclusive $I = 1$ hadronic τ decay width (non-strange branching fractions)

- HOWEVER, $\Gamma[\tau \rightarrow hadrons_{ud}\, \nu_\tau]$ alone ($\leftrightarrow w_{(00)}$ spectral integral with $s_0 = m_\tau^2$) insufficient to fix the 3 unknowns α_s, C_6, C_8

- ALEPH, OPAL approach
  - use $s_0 = m_\tau^2$, $(km) = (10), (11), (12), (13)$ "spectral weight" FESRs (FESRs with $w(y) \rightarrow w_{(km)}(y) = y^m (1-y)^k w_{(00)}(y)$) to supplement $ij = ud$ τ width
  - unsuppressed $D = 10, \cdots, 16$ contributions (each in principle present in *at least one* of the additional FESRs) assumed negligible *in all cases*
  - resulting five $s_0 = m_\tau^2$ spectral integrals used to fix 4 fit parameters, α_s, $\langle aG^2 \rangle$, C_6, C_8
  - NOTE: possible systematic error from neglect of $D > 8$ contributions NOT included in error estimates in the literature

Potential problems for the spectral weight analysis

- single s_0 $(= m_\tau^2)$ $\Rightarrow$ differing s_0-dependences of different D contributions not operative $\Rightarrow$ $D > 8$ effects (if non-negligible) absorbed into the fitted $D = 0, 4, 6, 8$ parameters *including* α_s

- ideal weight for α_s extraction *maximizes* $D = 0$, minimizes $D > 4$ contributions, whereas, as $k + m$ increases, (km) spectral weights have (see Table)
  - *increased* weighting of neglected $D > 8$ terms
  - *(rapidly) decreasing* $D = 0$ contributions

– Table: $\{b_m\}$, $r^{D=0}_{(km)}$ values for $w(y) = \sum_m b_m y^m$, with

* $I^D_{OPE}(w; s_0)$ the dimension D OPE integral

* $r^{D=0}_{(km)} = I^0_{OPE}(w_{(km)}; m_\tau^2)/I^0_{OPE}(w_{(00)}; m_\tau^2)$

(km)	$r^{D=0}_{(km)}$	$(b_2, b_3, b_4, b_5, b_6, b_7)$
(00)	1	$(-3, 2, 0, 0, 0, 0)$
(10)	0.721	$(-3, -5, -2, 0, 0, 0)$
(11)	0.154	$(-1, 3, 5, 2, 0, 0)$
(12)	0.059	$(\ 1, 1, -3, -5, -2, 0)$
(13)	0.027	$(\ 0, -1, -1, 3, 5, 2)$

Testing for actual problems in the spectral weight analysis

- If $D \leq 8$ parameters used in fit are compensating for neglected, but non-negligible, $D > 8$ contributions
  - $1/s_0^k$ scaling of $D = 2k + 2$ contributions $\Rightarrow s_0 = m_\tau^2$ fit quality will deteriorate with decreasing s_0
  - potential problems for fitted OPE when applied to FESRs involving independent weights of same degree (hence same $D \geq 4$ contributions) as those used in the fit

- Explicit tests in the V channel using the ALEPH best fit parameter results (V has the best fit quality among the V, A, V+A channels for the ALEPH analysis)

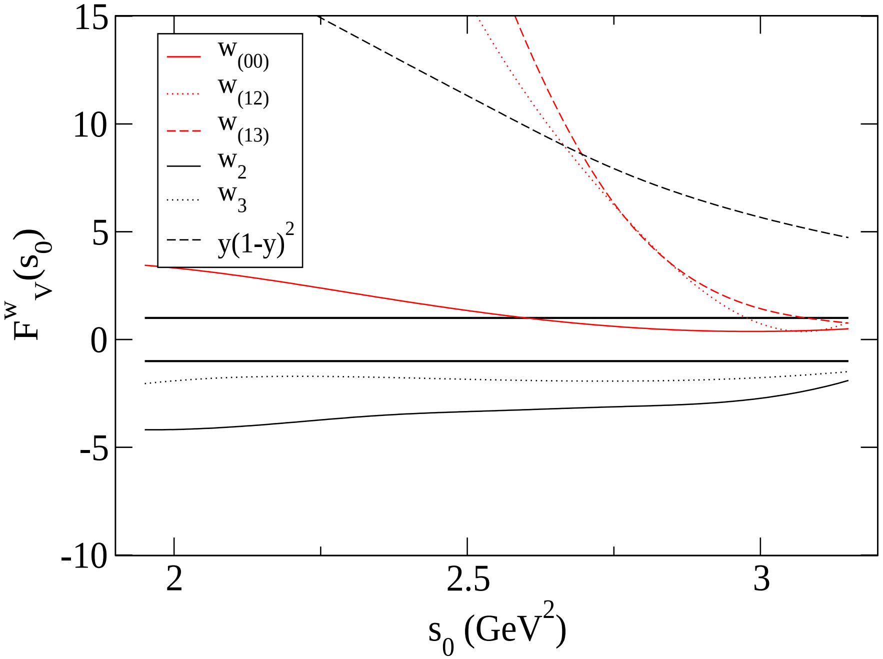
– with $I_{w;spec}(s_0) \pm \delta I_{w;spec}(s_0)$ the $w(y)$-weighted spectral integrals, define "fit qualities"

$$F^w_{V,A,V+A}(s_0) \equiv \left[I_{w;spec}(s_0) - I_{w;OPE}(s_0) \right] / \delta I_{w;spec}(s_0)$$

– FIGURE: $F^w_V(s_0)$ for ALEPH OPE fit values for

* a selection of the (km) spectral weights (red lines)
* 3 degree ≤ 3 weights, $w(y) = y(1-y)^2$, $w_2(y) = (1-y)^2$, $w_3(y) = 1 - \frac{3y}{2} + \frac{y^3}{2}$ (black lines)

- mismatch of spectral integrals, fitted OPE $\Rightarrow$ either a $D > 8$ problem or OPE breakdown: either way a problem for the extracted α_s



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$F^w_V(s_0)$
s_0 (GeV^2)
$w_{(00)}$
$w_{(12)}$
$w_{(13)}$
w_2
w_3
$y(1-y)^2$

A MODIFIED ANALYSIS

- Input:
  - ALEPH05 ud V,A,V+A data and covariances, supplemented by improved V/A separation for $\bar{K}K\pi$ (BaBar electroproduction $I = 0/1$ separation + CVC, as per Davier et al. [arXiv: 0803.0979])
  - alternate spectral input: OPAL ud V+A data and covariances
  - updated input for $D = 4$ condensates $\langle aG^2 \rangle$, $\langle \bar{s}s \rangle / \langle \bar{\ell}\ell \rangle$
  - FOPT, CIPT $D = 0$ treatments using new 5-loop Baikov, Chetyrkin, Kuhn [PRL101:012002] results, plus estimated 6-loop Adler function coefficient

- FESR choice: the weights $w_N(y) \equiv 1 - \frac{N}{N-1} y + \frac{1}{N-1} y^N$
  - as for $w_{(00)}(y)$, double zero at $s = s_0$ $(y = 1)$
  - a single unsuppressed $D > 4$ contribution for each w_N $(D = 2N + 2)$
  - integrated $D = 2N+2$ contribution scales as $1/s_0^{N+1}$ c.f. leading $D = 0$ term (good for fitting C_{2N+2})
  - modest *increase* of $I^{D=0}_{OPE}(w_N; s_0)$ with N (contrast spectral weight case)
  - increased $I^{D=2N+2}_{OPE}(w_N; s_0)$ suppression ($1/(N-1)$ w_N coefficient factor) with increasing N, hence further emphasis on $D = 0$, α_s

- Analysis proceedure
  - Simultaneous fit for $\alpha_s(m_\tau)$, C_{2N+2} using w_N FESR, $2.1\ \mathrm{GeV}^2 \lesssim s_0 < m_\tau^2$
  - Independent fit for each w_N
  - Independent fit (different C_D) for each of V, A, V+A channels
  - Independent fits using ALEPH and OPAL data sets
  - OPE uncertainties including $D = 0$ CIPT-FOPT difference

- Consistency checks/tests
  - consistency of $\alpha_s(m_\tau)$ from different w_N, V, A or V+A choice fixed
  - consistency of V, A, V+A $\alpha_s(m_\tau)$ determinations
  - ALEPH-based vs OPAL-based V+A results
  - s_0-dependence of w_N fit qualities $F^{w_N}_{V,A,V+A}(s_0)$
  - s_0-dependence of fit qualities for other $w(y)$, e.g., the degree ≤ 3 weights $w_{(00)}$, $y(1-y)^2$

RESULTS

- For the CIPT $D = 0$ treatment
  - OPAL-based V+A results

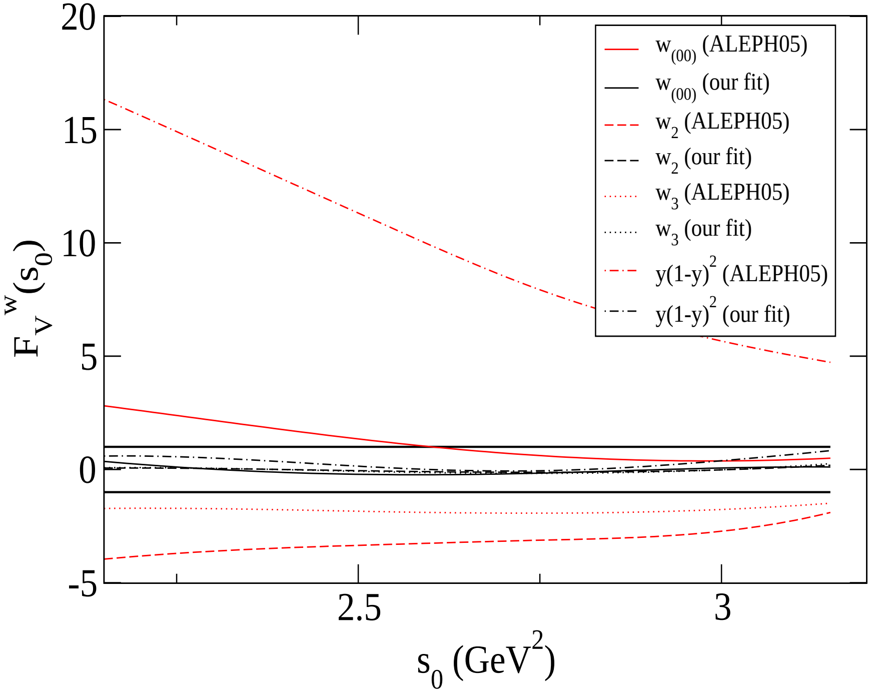
$w(y)$	$\alpha_s\left(m_\tau^2\right)$	C_{2N+2}/m_τ^{2N+2}
w_2	0.322(7)(12)	−0.000233(59)(114)
w_3	0.322(7)(12)	0.000205(74)(120)
w_4	0.322(7)(12)	−0.000162(76)(105)
w_5	0.322(7)(12)	0.000122(70)(86)
w_6	0.322(8)(12)	−0.000091(60)(67)

– ALEPH-based V, A and V+A results

Channel	$w(y)$	$\alpha_s\left(m_\tau^2\right)$	C_{2N+2}/m_τ^{2N+2}
V	w_2	0.321(7)(12)	−0.000187(29)(56)
	w_3	0.321(7)(12)	0.000060(36)(60)
	w_4	0.321(7)(12)	0.000015(36)(53)
	w_5	0.321(7)(12)	−0.000043(33)(44)
	w_6	0.321(7)(12)	0.000046(27)(35)
A	w_2	0.319(6)(12)	−0.000072(24)(60)
	w_3	0.319(6)(12)	0.000182(28)(71)
	w_4	0.319(6)(12)	−0.000216(27)(70)
	w_5	0.319(6)(12)	0.000201(23)(66)
	w_6	0.319(6)(12)	−0.000166(19)(59)
V+A	w_2	0.320(5)(12)	−0.000261(35)(114)
	w_3	0.320(5)(12)	0.000247(45)(125)
	w_4	0.320(5)(12)	−0.000208(44)(111)
	w_5	0.320(5)(12)	0.000166(39)(97)
	w_6	0.320(5)(12)	−0.000126(34)(88)

- NOTE much improved V, A, V+A consistency for ALEPH-based results

- Also see big improvement in s_0-dependent fit qualities
  - e.g., V channel degree ≤ 3 weight cases (FIGURE)
  - NOTE: improvement for $w_{(00)}$, $y(1-y)^2$, not just for weights used in fit
  - $|F^w_{V+A}(s_0)|$ even smaller (V+A: no need for V/A separation $\Rightarrow$ lower experimental errors $\Rightarrow$ favored channel for final α_s assessment)



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$F_V^w(s_0)$
s_0 (GeV^2)
$w_{(00)}$ (ALEPH05)
$w_{(00)}$ (our fit)
w_2 (ALEPH05)
w_2 (our fit)
w_3 (ALEPH05)
w_3 (our fit)
$y(1-y)^2$ (ALEPH05)
$y(1-y)^2$ (our fit)

- Features of the V+A fits (central α_s determination)
  - CIPT fit values consistent to ± 0.0001 across range $w_2, \cdots, w_6$ of weights
  - fitting small $D > 4$ contributions important to achieving this consistency (TABLE)
  - less consistency for FOPT (TABLE)
  - V+A fit results for $\alpha_s(m_\tau)$

$w(y)$	CIPT full fit	$s_0 = m_\tau^2$ CIPT $D > 4 \to 0$	FOPT full fit
w_2	0.320	0.310	0.320
w_3	0.320	0.316	0.315
w_4	0.320	0.319	0.313
w_5	0.320	0.321	0.312
w_6	0.320	0.322	0.312

– fitted $C_{D>4}$ show only modest accuracy (no surprise: weights chosen, by design, to suppress $D > 4$ contributions)

– Averaging ALEPH and OPAL based results with non-normalization component of error $\Rightarrow$

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.3209(46)_{exp}(118)_{th}$$

– standard self-consistent combination of 4-loop running, 3-loop matching at flavor thresholds $\Rightarrow$

$$\alpha_s^{(n_f=5)}(M_Z) = 0.1187(3)_{evol}(6)_{exp}(15)_{th}$$

– comparison with recent alternate determinations

Source	$\alpha_s^{(n_f=5)}(M_Z)$
Global EW fit	0.1191(27)
Shape observables in DIS, e^+e^-	0.1193($\sim$ 15)
R_{had} 2 $\rightarrow$ 10.6 GeV	$0.1190^{+0.0090}_{-0.0110}$
$\frac{\Gamma(\Upsilon(1s)\rightarrow\gamma X)}{\Gamma(\Upsilon(1s)\rightarrow X)}$	$0.1190^{+0.0060}_{-0.0050}$
lattice $\bar{c}c$ correlators	0.1174(12)
UV sensitive lattice observables	0.1192(11)
	0.1187(9)
This work	0.1187(16)

COMMENTS/CONCLUSIONS/OPINIONS

- $I = 1$ τ decay data yields one of the highest accuracy α_s determinations

- Careful treatment and fitting of $D > 4$ contributions mandatory to achieve this precision

- excellent agreement with the competing precision determination based on short-distance lattice observables

- contrast the $\sim 3\sigma$ discrepancy of a year ago, with 0.1170(12) from the lattice, 0.1215(12) from τ decays

- Re current errors:
  - theory error dominant ($\sim$ 2.5 times expt'l)
  - $D = 0$ truncation dominant theory error source (for $|FOPT - CIPT| \oplus O(a^5)$ estimate $\sim$ 0.010 of 0.012 total) $\Rightarrow$ main bottleneck for future improvements
  - Beneke-Jamin-like exploration (taking into account divergent nature of $D = 0$ series) crucial to any possibility of significant future error reduction

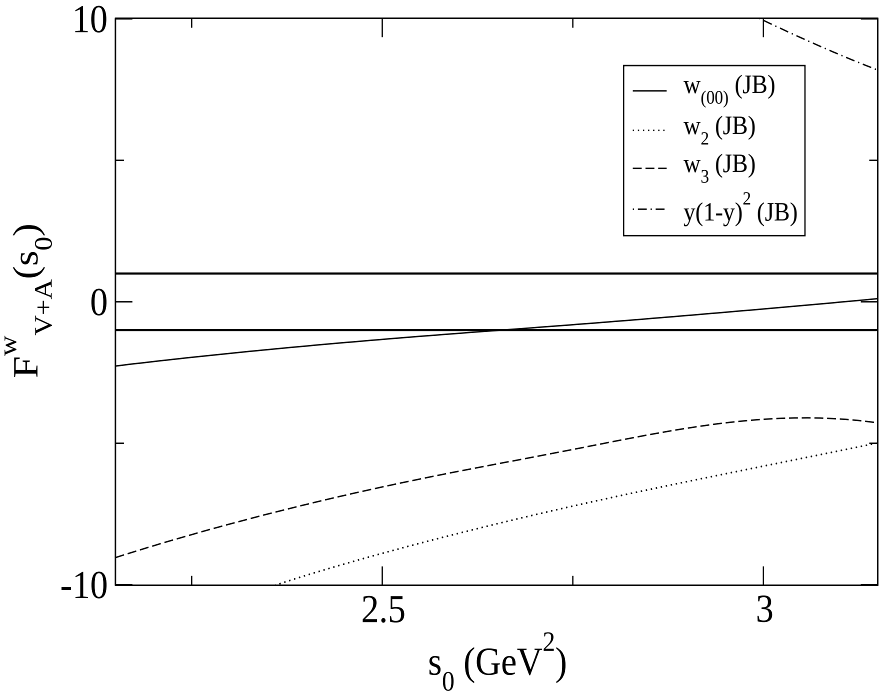
- Despite subleading contribution to errors, more experimental data (e.g., BaBar and Belle) desirable to weigh in on existing discrepancies in s-dependence of $dR_{ud;V+A}/ds$ (can impact fitted $D > 4$ coefficients)

SUPPLEMENTARY MATERIAL: Some observations on the Beneke-Jamin calculation

- As for the spectral weight analysis, control of $D > 4$ contributions essential for precision α_s (independent of choice of FOPT or CIPT for $D = 0$ contributions)

- Can test BJ input assumptions for $C_{6,8}$ for consistency with output FOPT fit α_s using $F^w_{V+A}(s_0)$ for various degree ≤ 3 $w(y)$ (FIGURE)

- Find problems for combination of assumed $D = 6, 8$ and FOPT fitted α_s



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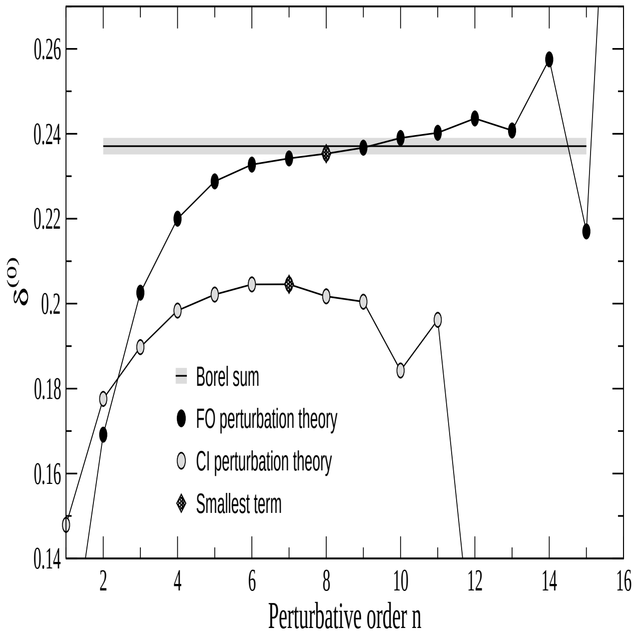
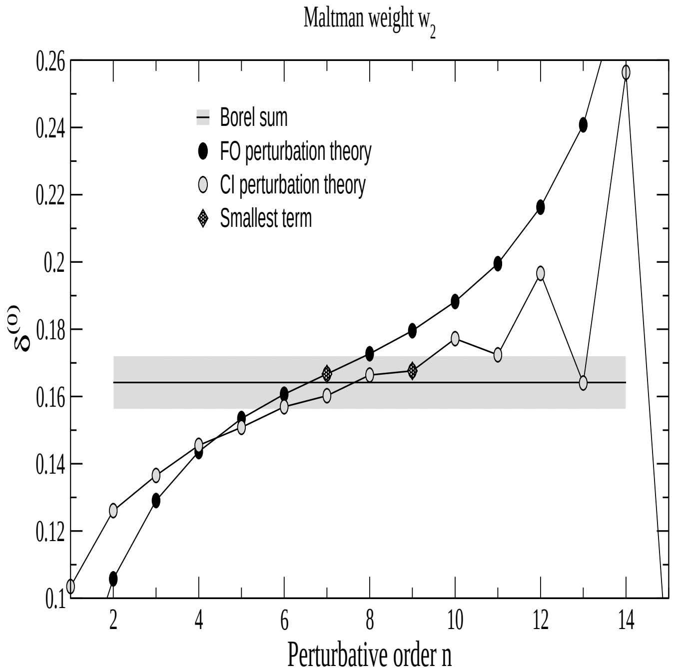
s_0 (GeV^2)
$w_{(00)}$ (JB)
w_2 (JB)
w_3 (JB)
$y(1-y)^2$ (JB)

- An exercise to test implications of the (minimal, 5-parameter) BJ model for the resummed $D = 0$ series
  - Features of the minimal model:
    - $*$ good approximation to full model sum using FOPT for a range of $w(y)$ (FIGURES)
    - $*$ CIPT approximation inferior to FOPT *most strongly so for* $w_{(0,0)}$ (FIGURES)
    - $*$ $\Rightarrow$ expect consistency of various FOPT fits, reduced consistency for CIPT fits

Beneke-Jamin model: FOPT, CIPT vs Borel sum

LEFT: $w_{(0,0)}$ RIGHT: w_2





- Test these expectations using combined w_2-w_3 fits for both FOPT, CIPT
  - ∗ the combined fits yield α_s, C_6, C_8, hence fix $w(y)$-weighted OPE integrals for any degree ≤ 3 $w(y)$
  - ∗ test agreement of CIPT, FOPT OPE versions with corresponding spectral integrals for $w_{(0,0)}$, $y(1-y)^2$
- test shows good consistency for CIPT; not so for FOPT (contrary to model expectations) (FIGURE)
- suggests alternate non-minimal modelling possible using such observations as constraints

FOPT vs CIPT w_2-w_3 joint fit V+A fit qualities