Chiral low-energy constants from τ data

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(M. G.-A., A. Pich & J. Prades, work in progress)

L_{10} & C_{87} : Motivation.

- ChPT Lagrangian:

$$\boxed{\begin{aligned}\mathfrak{L}^{\chi PT} = \frac{F^2}{4}\left\langle u_\mu u^\mu + \chi_+ \right\rangle + \ldots + \frac{1}{4} L_{10}^r \left\langle f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} \right\rangle + \ldots + \\ + \ldots + C_{87}^r \left\langle \nabla_\rho f_{-\mu\nu} \nabla^\rho f_-^{\mu\nu} \right\rangle + \ldots + O(p^8)\end{aligned}}$$

- The motivation is two-fold:
  - ☐ **Phenomenology**:

    more precise LEC's → more precise predictions.

    For example…

    $$\pi \to e \nu \gamma$$

  - ☐ **Theoretical**:

    Our estimation (directly from the data) allows us to test the quality of the different theoretical models that have predicted these LEC's.

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

- V-A correlator:

$$\Pi^{\mu\nu}_{V-A}(q) \equiv i \int d^4x\, e^{iqx} \langle 0 | T(J^{\mu}_{L}(x) J^{\nu}_{R}(0)^{\dagger}) | 0 \rangle$$
$$= (q^{\mu}q^{\nu} - q^2 g^{\mu\nu}) \Pi^{(0+1)}(q^2) + q^2 g^{\mu\nu} \Pi^{(0)}(q^2)$$

$$\boxed{\begin{aligned} J^{\mu}_{L} &= \overline{u}\gamma^{\mu}(1-\gamma_5)d \\ J^{\mu}_{R} &= \overline{u}\gamma^{\mu}(1+\gamma_5)d \end{aligned}}$$

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

- V-A correlator:

$$\begin{aligned}\Pi^{\mu\nu}_{V-A}(q) &\equiv i\int d^4x\, e^{iqx}\left\langle 0\right|T(J_L^\mu(x)J_R^\nu(0)^\dagger)\left|0\right\rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(0+1)}(q^2) + q^2 g^{\mu\nu}\Pi^{(0)}(q^2)\end{aligned}$$

$$\boxed{\begin{aligned} J_L^\mu &= \bar{u}\gamma^\mu(1-\gamma_5)d \\ J_R^\mu &= \bar{u}\gamma^\mu(1+\gamma_5)d \end{aligned}}$$

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- Weight function: $\underline{w(z)}\Pi(z)$

$$w(s) = \frac{1}{s}, \frac{1}{s^2}$$

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$$= (q^{\mu}q^{\nu} - q^2 g^{\mu\nu}) \boxed{\Pi^{(0+1)}(q^2)} + q^2 g^{\mu\nu}\Pi^{(0)}(q^2)$$

$$\boxed{\begin{aligned} J^{\mu}_{L} &= \bar{u}\gamma^{\mu}(1-\gamma_5)d \\ J^{\mu}_{R} &= \bar{u}\gamma^{\mu}(1+\gamma_5)d \end{aligned}}$$

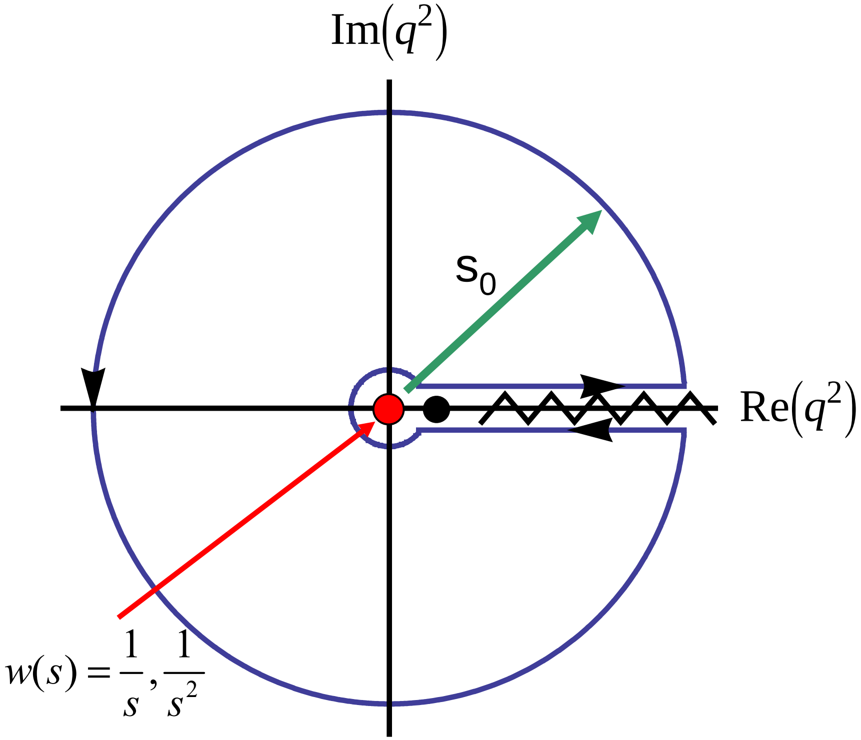
- Weight function: $\underline{w(z)}\Pi(z)$

$$w(s) = \frac{1}{s}, \frac{1}{s^2}$$

Analyticity of w(z)Π(z) relates different regions of the C-plane. → **SUM RULE**

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

Analytic properties of $w(z)\Pi(z)$



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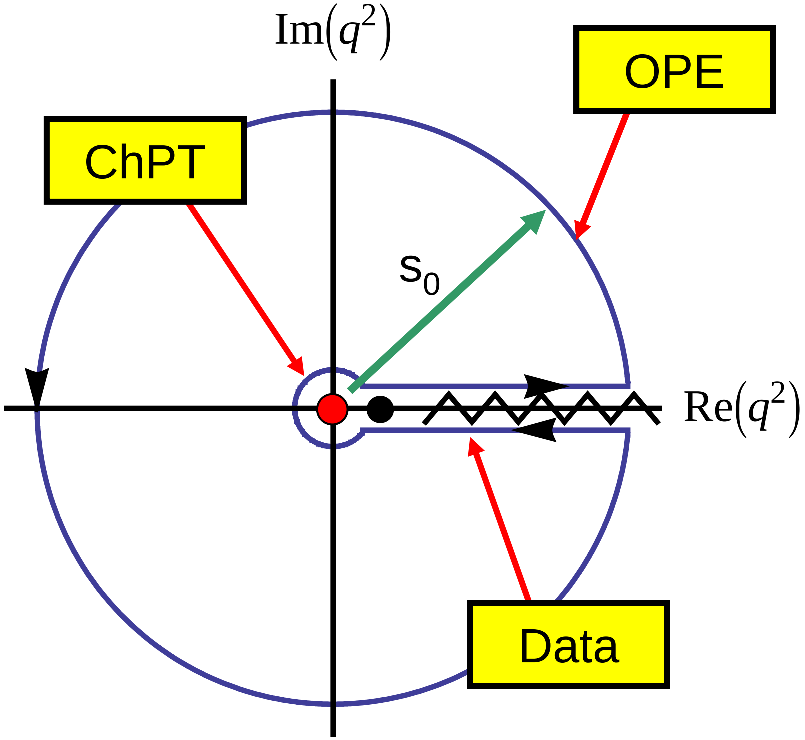

V-A spectral function

$$\operatorname*{Res}_{z=0}\frac{\Pi^{OPE}(z)}{z^n}-\operatorname*{Res}_{z=0}\frac{\Pi^{\chi PT}(z)}{z^n}=\frac{2F_\pi^2}{\left(m_\pi^2\right)^n}-\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

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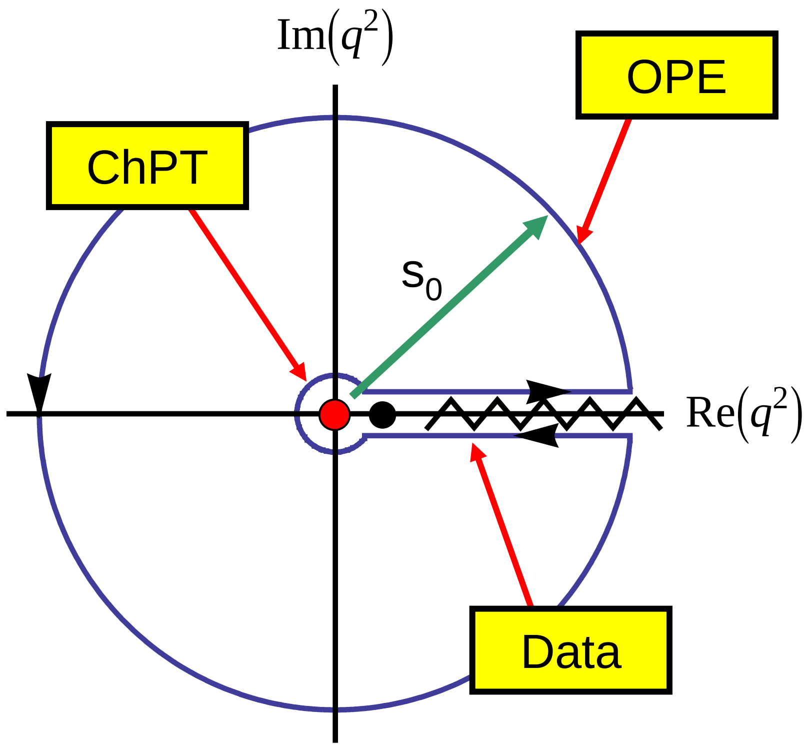

$$\cancel{\underset{z=0}{\text{Res}}\frac{\Pi^{OPE}(z)}{z^n}}-\underset{z=0}{\text{Res}}\frac{\Pi^{\chi PT}(z)}{z^n}=\frac{2F_\pi^2}{\left(m_\pi^2\right)^n}-\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

$$\frac{d^{n-1}}{dz^{n-1}}\Pi^{\chi PT}(z)-\frac{2F_\pi^2}{\left(m_\pi^2\right)^n}=\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

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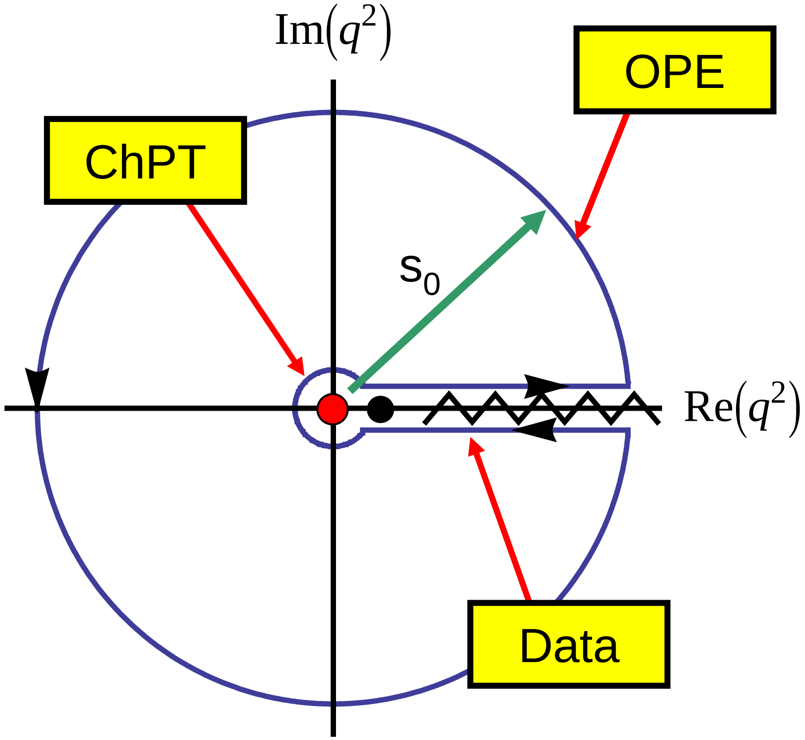

$$\cancel{\underset{z=0}{\mathrm{Res}}\frac{\Pi^{OPE}(z)}{z^n}} - \underset{z=0}{\mathrm{Res}}\frac{\Pi^{\chi PT}(z)}{z^n} = \frac{2F_\pi^2}{\left(m_\pi^2\right)^n} - \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

$$\frac{d^{n-1}}{dz^{n-1}}\Pi^{\chi PT}(z) - \frac{2F_\pi^2}{\left(m_\pi^2\right)^n} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

$$\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds$$

$$\frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds$$

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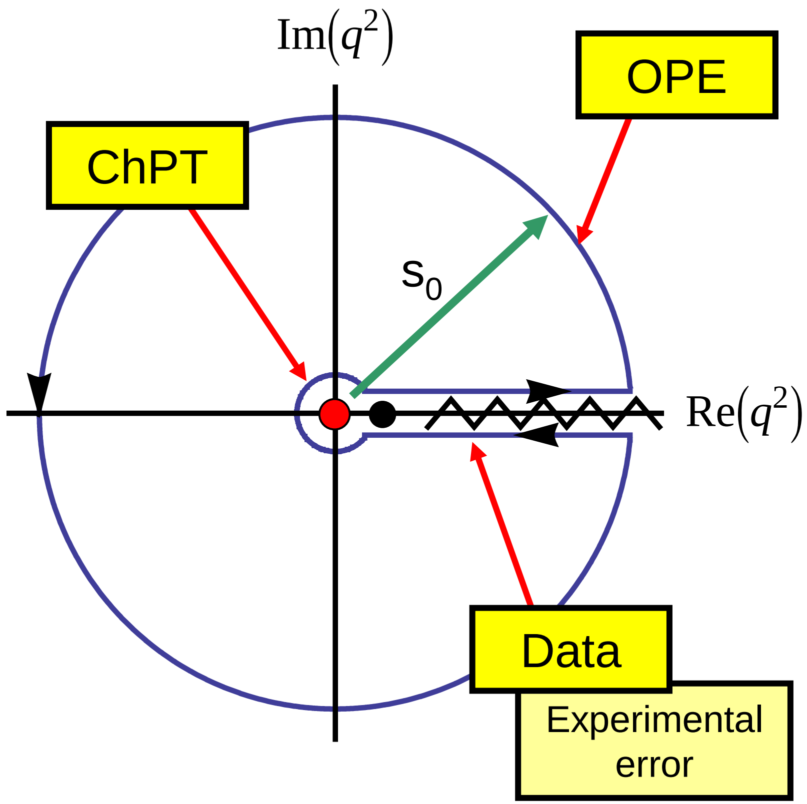

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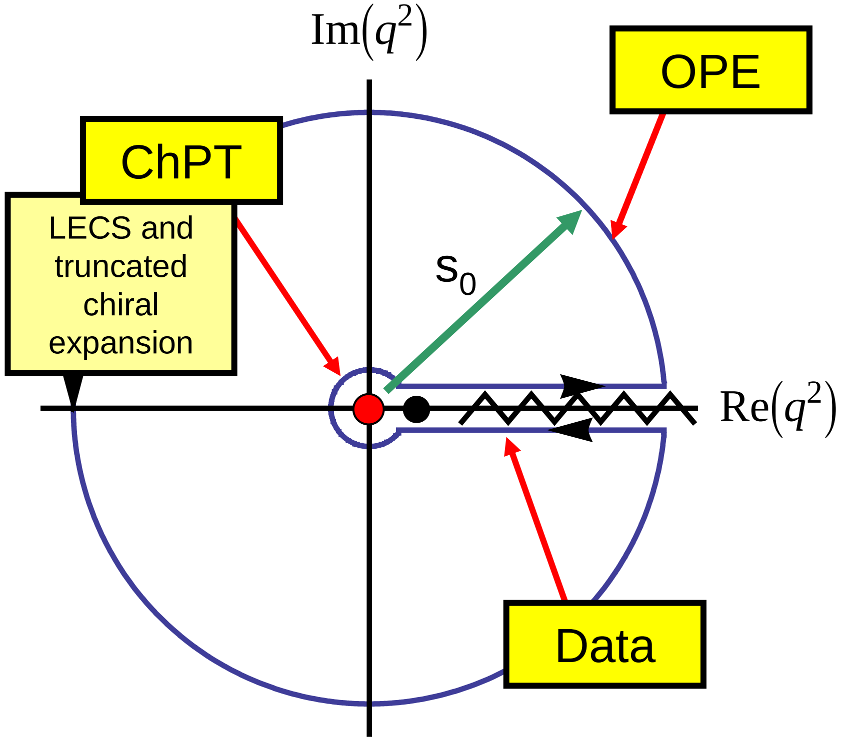

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$$\boxed{\begin{aligned}&\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds\\ &\frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds\end{aligned}}$$

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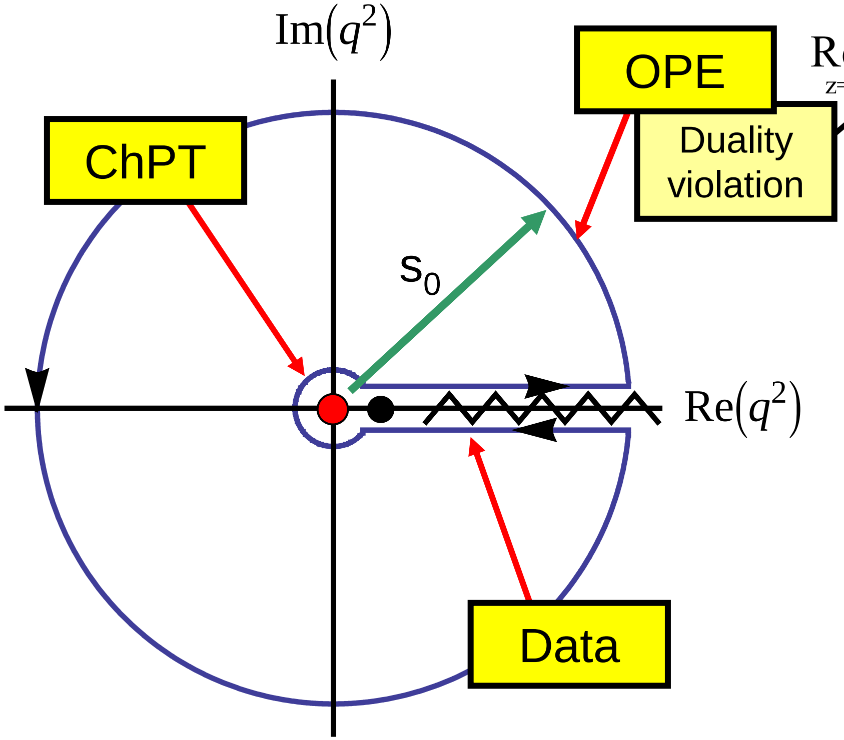

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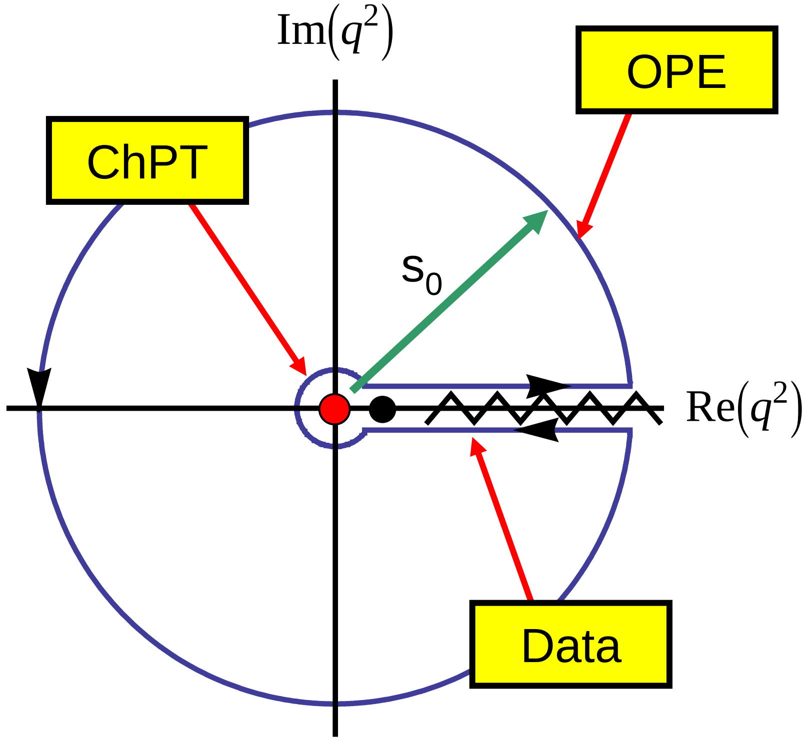

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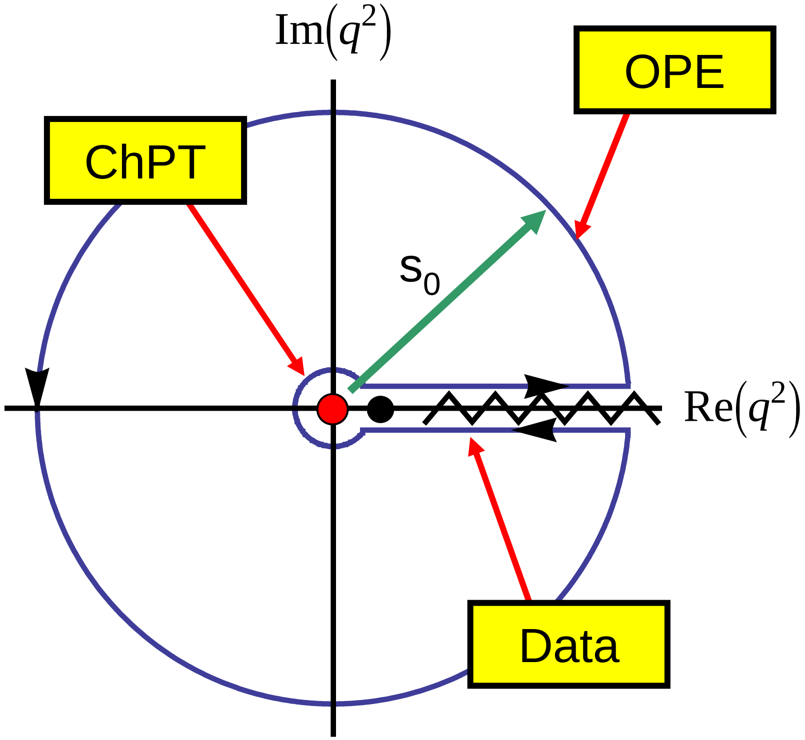
$$\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds \equiv -8L_{10}^{eff}$$

$$\frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds \equiv 16C_{87}^{eff}$$

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

Analytic properties of $w(z)\Pi(z)$



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$$\underbrace{\frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4}}_{\chi\text{PT side}} = \underbrace{\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds \equiv 16C_{87}^{eff}}_{\text{Data side}}$$

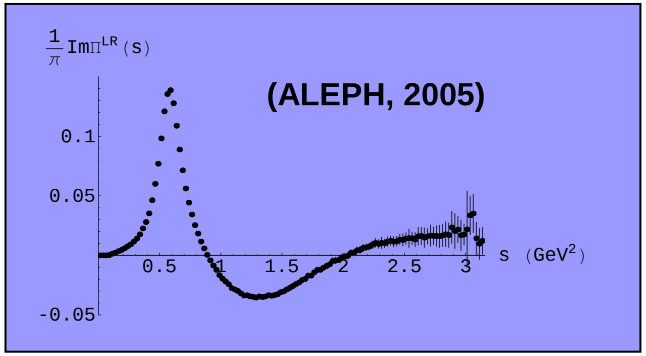
Data side

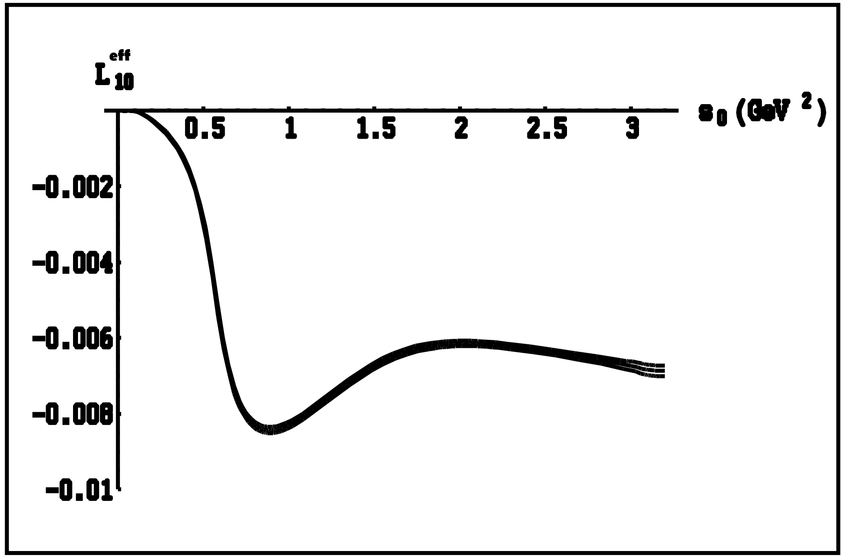
- Eff. parameters: determination.

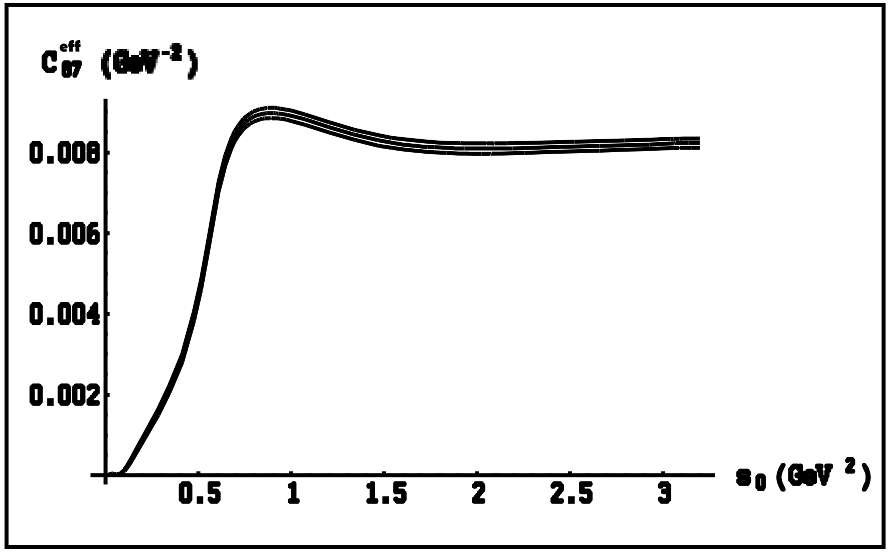
$$-8L_{10}^{eff} \equiv \int_{s_{th}}^{s_0} \frac{\rho(s)}{s} ds$$

$$16C_{87}^{eff} \equiv \int_{s_{th}}^{s_0} \frac{\rho(s)}{s^2} ds$$

We can measure ρ(s) from the hadronic tau decays…

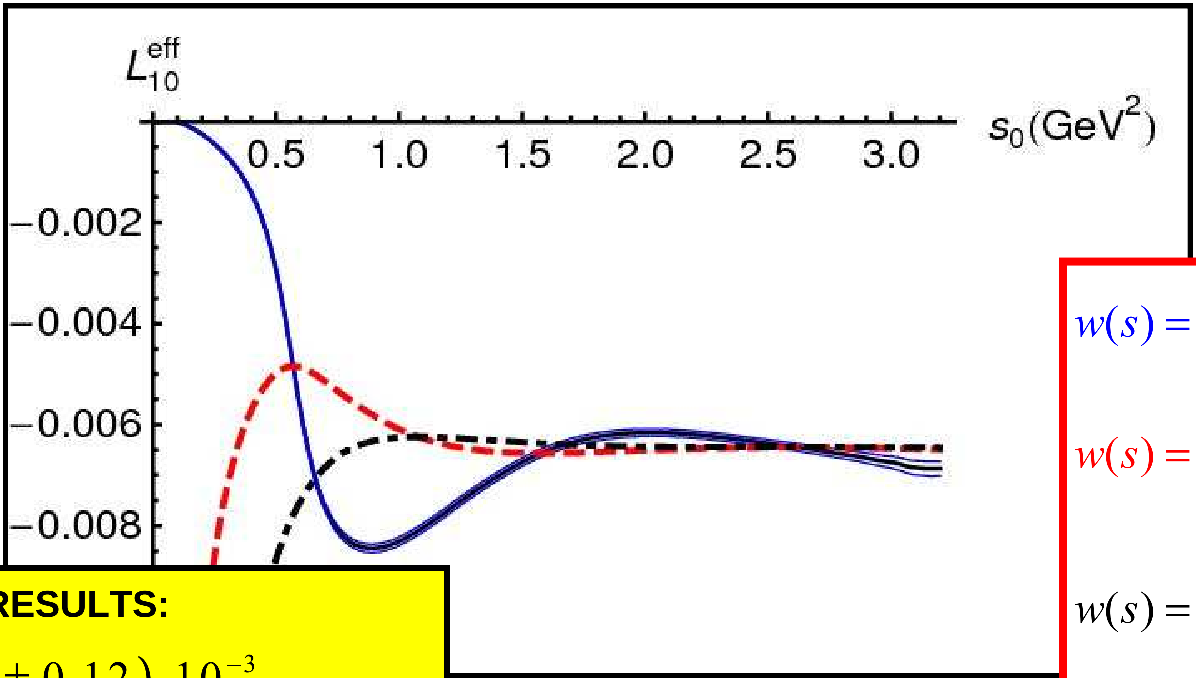






Data side

- Eff. parameters: determination.



$$w(s)=\frac{1}{s}$$

$$w(s)=\frac{1}{s}\left(1-\frac{s}{s_0}\right)$$

$$w(s)=\frac{1}{s}\left(1-\frac{s}{s_0}\right)^2$$

RESULTS:

$$L_{10}^{eff}=-\left(6.48\pm0.12\right)\cdot10^{-3}$$

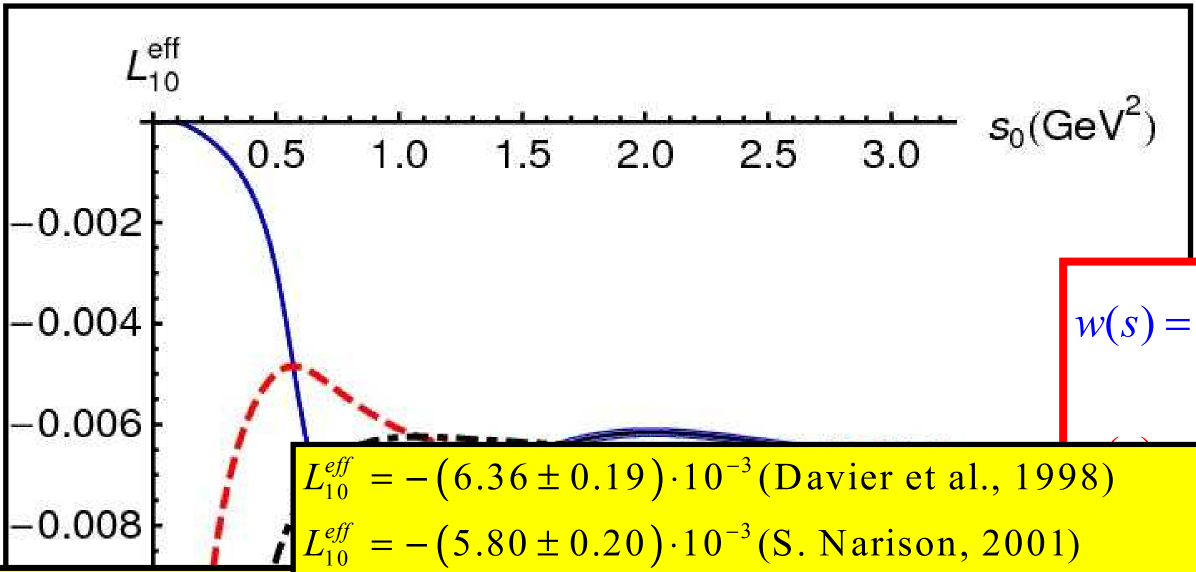
$$C_{87}^{eff}=+\left(8.18\pm0.14\right)\cdot10^{-3}\,\mathrm{GeV}^{-2}$$

Good accuracy!

(F. Le Diberder & A. Pich, 1992,
K. Maltman, 1998,
Domínguez & Schilcher, 1999, ...)

Data side

- Eff. parameters: determination.



$$w(s) = \frac{1}{s}$$

$$L_{10}^{eff} = -(6.36 \pm 0.19)\cdot 10^{-3} \text{(Davier et al., 1998)}$$
$$L_{10}^{eff} = -(5.80 \pm 0.20)\cdot 10^{-3} \text{(S. Narison, 2001)}$$
$$L_{10}^{eff} = -(6.45 \pm 0.06)\cdot 10^{-3} \text{(Domínguez et al., 2006)}$$
$$C_{87}^{eff} = +(8.10 \pm 0.12)\cdot 10^{-3}\,\text{GeV}^{-2} \text{ (Domínguez \& Schilcher, 2004)}$$

RESULTS:

$$L_{10}^{eff} = -(6.48 \pm 0.12)\cdot 10^{-3}$$
$$C_{87}^{eff} = +(8.18 \pm 0.14)\cdot 10^{-3}\,\text{GeV}^{-2}$$

Good accuracy!

(F. Le Diberder & A. Pich, 1992,
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χPT side

- Theoretical calculation of the effective parameters

$$\boxed{\begin{aligned} L_{10}^{eff} &= \frac{-1}{8}\left(\Pi^{\chi PT}(0)+2\frac{F_\pi^2}{m_\pi^2}\right)\equiv\frac{-1}{8}\overline{\Pi}^{\chi PT}(0)=f(L_{10}^r(\mu),\mu) \\ C_{87}^{eff} &= \frac{1}{16}\left(\Pi^{\chi PT}{}'(0)+2\frac{F_\pi^2}{m_\pi^4}\right)\equiv\frac{1}{16}\overline{\Pi}^{\chi PT}(0)=g(C_{87}^r(\mu),\mu) \end{aligned}}$$

- Using ChPT…

$$\overline{\Pi}^{\chi PT}(s)=-8L_{10}^r-8B_V^{\pi\pi}(s)-4B_V^{KK}(s)+O(p^6)$$

χPT side

- Theoretical calculation of the effective parameters

$$\boxed{\begin{aligned} L_{10}^{eff} &= \frac{-1}{8}\left(\Pi^{\chi PT}(0)+2\frac{F_\pi^2}{m_\pi^2}\right) \equiv \frac{-1}{8}\overline{\Pi}^{\chi PT}(0) = f(L_{10}^r(\mu),\mu) \\ C_{87}^{eff} &= \frac{1}{16}\left(\Pi^{\chi PT}\,{}'(0)+2\frac{F_\pi^2}{m_\pi^4}\right) \equiv \frac{1}{16}\overline{\Pi}^{\chi PT}(0) = g(C_{87}^r(\mu),\mu) \end{aligned}}$$

- Using ChPT…

$$\begin{aligned} \overline{\Pi}^{\chi PT}(s) = {} & -8L_{10}^r - 8B_V^{\pi\pi}(s) - 4B_V^{KK}(s) \\ & +16C_{87}^r s - 32m_\pi^2\left(C_{61}^r - C_{12}^r - C_{80}^r\right) - 32\left(m_\pi^2 + 2m_K^2\right)\left(C_{62}^r - C_{13}^r - C_{81}^r\right) \\ & +\frac{16}{f_\pi^2}\left(\left(2\mu_\pi + \mu_\pi\right)\left(L_9^r + 2L_{10}^r\right) - \left(2B_V^{\pi\pi}(s) + B_V^{KK}(s)\right)sL_9^r\right) \\ & +G_{2L}(s,\mu) + O(p^8) \end{aligned}$$

(Amorós et al., 2000)

(Golowich & Kambor, 1995, 1998)

χPT side

- Theoretical calculation of the effective parameters

$$L_{10}^{eff} = \frac{-1}{8}\left(\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2}\right) \equiv \frac{-1}{8}\bar{\Pi}^{\chi PT}(0) = f(L_{10}^r(\mu), \mu)$$

$$C_{87}^{eff} = \frac{1}{16}\left(\Pi^{\chi PT}{}'(0) + 2\frac{F_\pi^2}{m_\pi^4}\right) \equiv \frac{1}{16}\bar{\Pi}^{\chi PT}(0) = g(C_{87}^r(\mu), \mu)$$

- Therefore at O(p^4)…

$$L_{10}^{eff} = \tfrac{-1}{8}\bar{\Pi}^{\chi PT}(0) = L_{10}^r(\mu) + \left(\frac{\log\frac{m_K^2}{m_\pi^2}}{384\pi^2} + \frac{1+\log\frac{m_\pi^2}{\mu^2}}{128\pi^2}\right)$$

$$L_{10}^r(m_\rho) = -(5.22 \pm 0.12)\cdot 10^{-3}$$

$$L_{10}^r(m_\rho) = -(5.13 \pm 0.19)\cdot 10^{-3}$$

(Davier et al., 1998)

χPT side

$$L_{10}^{eff}=\frac{-1}{8}\overline{\Pi}^{\chi PT}(0)$$

$$C_{87}^{eff}=\frac{1}{16}\overline{\Pi}^{\chi PT}(0)$$

- At O(p^6)…

$$\begin{aligned}L_{10}^{eff}=L_{10}^{r}+&\left(\frac{\log\frac{m_K^2}{m_\pi^2}}{384\pi^2}+\frac{1+\log\frac{m_\pi^2}{\mu^2}}{128\pi^2}\right)+\\&+4m_\pi^2\left(C_{61}^r-C_{12}^r-C_{80}^r\right)+4\left(m_\pi^2+2m_K^2\right)\left(C_{62}^r-C_{13}^r-C_{81}^r\right)\\&-\frac{2}{f_\pi^2}(2\mu_\pi+\mu_\pi)\left(L_9^r+2L_{10}^r\right)\\&-\frac{1}{8}G_{2L}(0,\mu)\end{aligned}$$

$$\begin{aligned}C_{87}^{eff}=&\frac{1}{7680\pi^2}\left(\frac{1}{m_K^2}+\frac{2}{m_\pi^2}\right)+\\&+C_{87}^r(\mu)\\&-\frac{2L_9^r(\mu)}{f_\pi^2}\left(\frac{\log\frac{m_K^2}{m_\pi^2}}{384\pi^2f_\pi^2}+\frac{1+\log\frac{m_\pi^2}{\mu^2}}{128\pi^2}\right)\\&+\frac{1}{16}G'_{2L}(0,\mu)\end{aligned}$$

χPT side

$$L_{10}^{eff} = \frac{-1}{8}\overline{\Pi}^{\chi PT}(0)$$

$$C_{87}^{eff} = \frac{1}{16}\overline{\Pi}^{\chi PT}(0)$$

- At $O(p^6)$…

$$L_{10}^{eff} = L_{10}^{r} + \left(\frac{\log \frac{m_K^2}{m_\pi^2}}{384\pi^2} + \frac{1+\log \frac{m_\pi^2}{\mu^2}}{128\pi^2} \right) + 4m_\pi^2\left(C_{61}^r - C_{12}^r - C_{80}^r\right) + 4\left(m_\pi^2 + 2m_K^2\right)\left(C_{62}^r - C_{13}^r - C_{81}^r\right) - \frac{2}{f_\pi^2}(2\mu_\pi + \mu_\pi)\left(L_9^r + 2L_{10}^r\right) - \frac{1}{8}G_{2L}(0,\mu)$$

$$C_{87}^{eff} = \frac{1}{7680\pi^2}\left(\frac{1}{m_K^2} + \frac{2}{m_\pi^2}\right) + C_{87}^r(\mu) - \frac{2L_9^r(\mu)}{f_\pi^2}\left(\frac{\log \frac{m_K^2}{m_\pi^2}}{384\pi^2 f_\pi^2} + \frac{1+\log \frac{m_\pi^2}{\mu^2}}{128\pi^2} \right) + \frac{1}{16}G'_{2L}(0,\mu)$$

Experimental error

LEC's error

$$L_{10}^r(m_\rho) = -\left(4.06 \pm 0.08 \pm 0.39\right)\cdot 10^{-3} = -\left(4.06 \pm 0.40\right)\cdot 10^{-3}$$

$$C_{87}^r(m_\rho) = +\left(4.89 \pm 0.14 \pm 0.13\right)\cdot 10^{-3}\,\mathrm{GeV}^2 = +\left(4.89 \pm 0.19\right)\cdot 10^{-3}\,\mathrm{GeV}^2$$

Comparisons

- L_{10}: extractions from the data:

$$L_{10}^r(m_\rho) = -\left(4.06 \pm 0.40\right)\cdot 10^{-3}$$

$$L_{10}^r(m_\rho) = -\left(5.13 \pm 0.19\right)\cdot 10^{-3} \text{(From inclusive } \tau \text{ data, Davier et al., 1998)}$$

$$L_{10}^r(m_\rho) = -\left(4.5 \pm 0.4\right)\cdot 10^{-3} \text{(From } \pi \to e\nu\gamma \text{ \& } \left\langle r^2 \right\rangle_\pi \text{, 2008)}$$

- L_{10}: theoretical estimations:

µ-sensitive

(NLO calculation)

$$L_{10}^r(m_\rho) = -\left(4.4 \pm 0.9\right)\cdot 10^{-3} \text{(Pich et al., 2008)}$$

Not µ-sensitive

(LO calculation)

$$L_{10}^r(m_\rho) = -\left(5.7 \pm 1.4\right)\cdot 10^{-3} \text{(Ecker et al., 1989)}$$

(updated)

25% assigned error

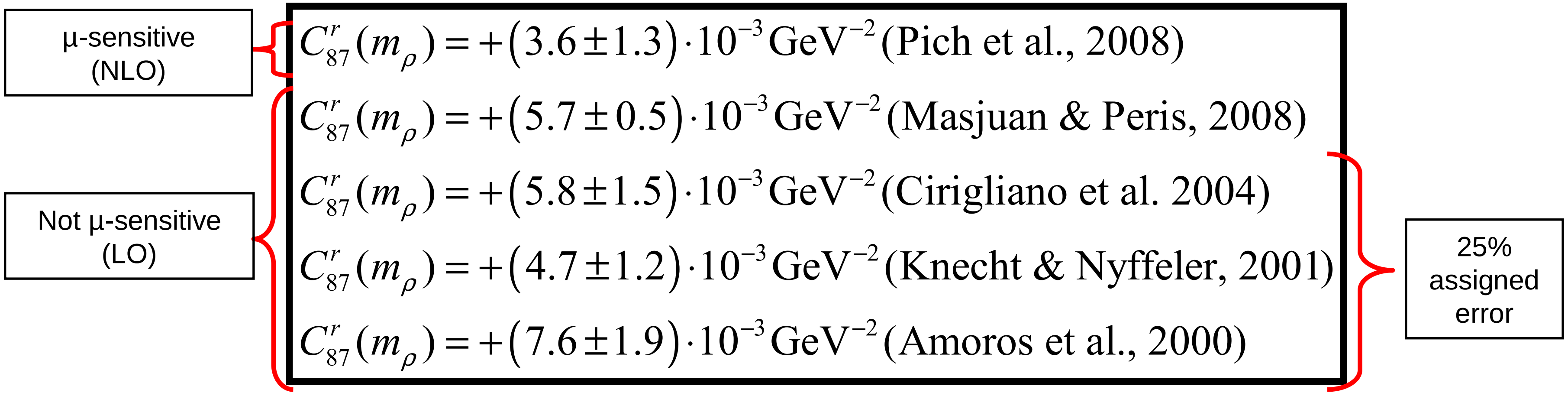
Lattice…

$$L_{10}^r(m_\rho) = -\left(5.2 \pm 0.5\right)\cdot 10^{-3} \text{(Shintani et al., 2008)}$$

Comparisons

- C_{87}: With theoretical (model dependent) estimations:

$$\boxed{C_{87}^{r}(m_\rho) = +\left(4.89 \pm 0.19\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}}$$



μ-sensitive (NLO)

$$C_{87}^{r}(m_\rho) = +\left(3.6 \pm 1.3\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}\,(\text{Pich et al., 2008})$$

Not μ-sensitive (LO)

$$C_{87}^{r}(m_\rho) = +\left(5.7 \pm 0.5\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}\,(\text{Masjuan \& Peris, 2008})$$

$$C_{87}^{r}(m_\rho) = +\left(5.8 \pm 1.5\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}\,(\text{Cirigliano et al. 2004})$$

$$C_{87}^{r}(m_\rho) = +\left(4.7 \pm 1.2\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}\,(\text{Knecht \& Nyffeler, 2001})$$

$$C_{87}^{r}(m_\rho) = +\left(7.6 \pm 1.9\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}\,(\text{Amoros et al., 2000})$$

25% assigned error

$L_9^r, \bar{l}_5$ and $\bar{l}_6$

- SU(3) χPT LECs $\overset{\mathrm{O}(p^6)}{\Longleftrightarrow}$ SU(2) χPT LECs (J. Gasser et al., 2007)

$$\boxed{L_{10}^r(m_\rho) \quad \Rightarrow \quad \bar{l}_5 = +12.24 \pm 0.29 \quad \leftarrow \mathrm{O}(p^6)}$$

- L$_9$ & l$_6$ determination:

$$L_9^r(m_\rho) = +(5.93 \pm 0.43) \cdot 10^{-3}$$

OK!

(From $\langle r^2 \rangle_\pi$, Bijnens & Talavera, 2002)

$$\boxed{\begin{array}{lll} (L_9^r + L_{10}^r)(m_\rho) = -(1.44 \pm 0.08) \cdot 10^{-3} & \Rightarrow L_9^r(m_\rho) = +(5.50 \pm 0.41) \cdot 10^{-3} & \leftarrow \mathrm{O}(p^6) \\ \bar{l}_6 - \bar{l}_5 = +(2.98 \pm 0.33) \cdot 10^{-3} & \Rightarrow \bar{l}_6 = +15.22 \pm 0.44 & \leftarrow \mathrm{O}(p^6) \end{array}}$$

(From $\pi \to e\nu\gamma$,
J. Bijens & P. Talavera, 1997,
R. Unterdorfer & H. Pichl, 2008)

Conclusions

- We have calculated L_{10} with recent inclusive tau data including $O(p^6)$ terms in the chiral expansion, obtaining

$$\boxed{L_{10}^r(m_\rho) = -\left(4.06 \pm 0.40\right) \cdot 10^{-3}} \Longrightarrow \boxed{\begin{aligned} L_9^r(m_\rho) &= +\left(5.50 \pm 0.41\right) \cdot 10^{-3} \\ \bar{l}_5 &= +12.24 \pm 0.29 \\ \bar{l}_6 &= +15.22 \pm 0.44 \end{aligned}}$$

- The same approach has allowed us to calculate the first determination from data of C_{87} (up to $O(p^6)$ terms) obtaining

$$\boxed{C_{87}^r(m_\rho) = +\left(4.89 \pm 0.19\right) \cdot 10^{-3}\,\mathrm{GeV}^{-2}}$$

… in good agreement with different theoretical predictions.

Backup

Operator Product Expansion

(Wilson, 1967):

$$\Pi_{V-A}(q^2) = \Pi_{V-A}^{pert}(q^2) + \frac{\mathcal{O}_2}{-q^2} + \frac{\mathcal{O}_4}{(-q^2)^2} + \frac{\mathcal{O}_6}{(-q^2)^3} + \ldots$$

The ChPT Lagrangian

$$\begin{aligned}\mathfrak{L}^{\chi PT} &= \frac{F^2}{4}\left\langle u_\mu u^\mu + \chi_+ \right\rangle + \\ &+\ldots+\frac{1}{4}L_{10}\left\langle f_{+\mu\nu}f_+^{\mu\nu} - f_{-\mu\nu}f_-^{\mu\nu}\right\rangle+\ldots+ \\ &+\ldots+C_{87}\left\langle \nabla_\rho f_{-\mu\nu}\nabla^\rho f_-^{\mu\nu}\right\rangle+\ldots+O(p^8)\end{aligned}$$

$$u_\mu = i\left[u^\dagger\left(\partial_\mu - i r_\mu\right)u - u\left(\partial_\mu - i l_\mu\right)u^\dagger\right]$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$$

Meson fields...

$$u(\phi) = e^{\frac{i}{\sqrt{2}F}\phi}$$

$$\phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\phi_i = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

External fields...

$$\chi = 2\mathrm{B}_0\left(s + ip\right)$$

$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]$$

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]$$

The explicit breaking of chiral symmetry through quark masses can be added by...

$$s = \mathfrak{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$