CURRENT STATUS OF THE EXTRACTION OF V_{us} FROM HADRONIC τ DECAY

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OUTLINE

- *Some relevant basics*
- *Technical issues/questions*
- *Results and prospects*

BACKGROUND/NOTATION/TERMINOLOGY

- V,A $ij = ud,\ us$, $(J) = (0+1),\ (0)$ spectral functions from experimental differential decay distributions

$$R_{V/A;ij} = 12\pi^2\, |V_{ij}|^2 S_{EW} \int_0^1 dy_\tau \left[w_T(y_\tau)\, \rho^{(0+1)}_{V/A;ij}(s) + w_L(y_\tau)\, \rho^{(0)}_{V/A;ij}(s) \right]$$

with $R_{V/A;ij} \equiv \frac{\Gamma[\tau \to \nu_\tau \, \text{hadrons}_{V/A;ij}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$, $y_\tau = s/m_\tau^2$

$w_T(y) = (1-y)^2(1+2y)$, $w_L(y) = -2y(1-y)^2$,

- $\Rightarrow$ linear combination of the spectral functions $\rho^{(0+1)}_{V/A;ij}(s)$ and $\rho^{(0)}_{V/A;ij}(s)$ from $dR_{V/A;ij}/ds$

- "longitudinal": pure $(J) = (0)$ term

- "(k, m) spectral weights": experimental distribution multiplied by $(1 - y_\tau)^k \, y_\tau^m$ before integration

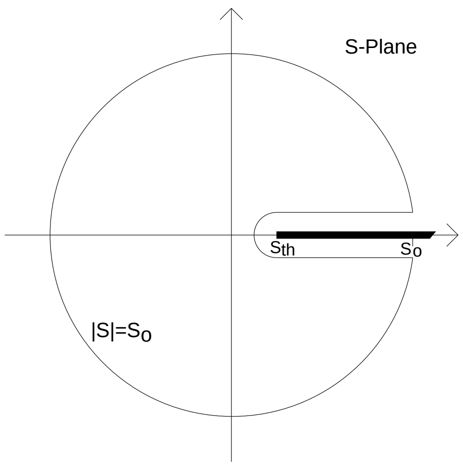
- $(0, 0)$ spectral weight: kinematic weight case $\Rightarrow R^{(0,0)}_{V/A;ij}$ from sum over branching fractions

EXTRACTING V_{us}

- Basic FESR relation for kinematic-singularity-free $\Pi(s)$

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

[LHS: data; RHS: OPE]



- $R^w_{ij}(s_0)$: generic $(J) = (0+1)$ or (0); V, A or V+A; $w(s)$-weighted $0 < s \leq s_0 \leq m_\tau^2$ spectral integral

- V_{us} (and m_s) from flavor-breaking combinations

$$\delta R^w(s_0) = \left[R^w_{ud}(s_0)/|V_{ud}^2|\right] - \left[R^w_{us}(s_0)/|V_{us}^2|\right]$$

  - $[\delta R^w(s_0)]^{OPE}_{D=0} = 0$ *(for physical V_{us} only)*

  - incorrect $V_{us} \Rightarrow$ residual (large) $D = 0$ OPE contribution, hence leverage for determining V_{us}

    [Gamiz et al., JHEP 0301: 060 (2003)]

  - (Leading OPE contribution, $[\delta R^w(s_0)]^{OPE}_{D=2} \propto m_s^2 \Rightarrow$ joint fit for V_{us}, m_s also possible)

- With input m_s from other sources, V_{us} via

$$|V_{us}| = \sqrt{R^w_{us}(s_0) / \left[\frac{R^w_{ud}(s_0)}{|V^2_{ud}|} - \delta R^w_{OPE}(s_0) \right]}$$

  – KEY POINT: $R^w_{ud}(s_0)$ typically $>>$ $\delta R^w_{OPE}(s_0) \Rightarrow$

    * fractional OPE-induced error on $V_{us} \sim \frac{\delta R^w_{OPE}(s_0)}{2R^w_{ud}(s_0)}$ $<<$ that on $\delta R^w_{OPE}(s_0)$ itself

    * $\Rightarrow$ good precision V_{us} from modest precision OPE

  – pre-2007 data errors ($3-4\%$ on R^w_{us} $\Rightarrow$ $1.5-2\%$ on $|V_{us}|$), reductions in progress by Belle, BaBar (currently 2.4% on $B_{us;TOT}$)

COMPLICATIONS FOR DETERMINING $\delta R^w_{OPE}(s_0)$

- *severe* problems with longitudinal OPE representation and need for longitudinal subtraction/phenomenological modelling
- problems associated with slow convergence of integrated $(0+1)$ $D=2$ OPE series
- poorly constrained/unknown higher dimension $(D>4)$ OPE contributions

PROBLEMS WITH THE LONGITUDINAL OPE

- integrated longitudinal $D = 2$ OPE series badly non-convergent for all kinematically-allowed scales

- (even worse) *ALL* truncation schemes employed in the literature *BADLY* violate longitudinal continuum spectral positivity [KM, J. Kambor, PRD64: 093014]

- $\Rightarrow$ *phenomenological input for longitudinal spectral contributions MANDATORY (equivalently: subtract longitudinal contributions and work with* $(0+1)$ *sum rules)*

- THE LONGITUDINAL SUBTRACTION

  - K, π pole terms very accurately known, dominant for chiral+kinematic reasons

  - residual us "continuum" subtraction

    * Jamin, Oller, Pich dispersive analysis for us scalar

    * KM, J. Kambor sum rule analysis for us PS

    * *both determinations strongly constrained by implications for m_s from scalar, PS sum rules*

  - numerical impact small: $\sim$ 20% uncertainty on continuum subtraction $\leftrightarrow \delta|V_{us}| \sim 0.0002$

SLOW $D = 2$ $(0+1)$ OPE SERIES CONVERGENCE

- $\Delta\Pi(Q^2) \equiv \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$, $\Delta\rho(s)$: correlator and corresponding spectral function difference appearing in $\delta R^{w,(0+1)}(s_0)$, $\delta R^{w,(0+1)}_{OPE}(s_0)$

- $D = 2$ OPE series, $\bar{m}_s = m_s(Q^2)$, $\bar{a} = \alpha_s(Q^2)/\pi$, $\overline{MS}$ scheme [Baikov, Chetyrkin, Kuhn PRL95:012003]

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\Big[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \cdots\Big]$$

- $a(m_\tau^2) \sim 0.10 - 0.11$ hence series *very* slowly converging at spacelike point on $|s| = s_0 = m_\tau^2$

- SLOW $D = 2$ (0+1) CONVERGENCE OPTIONS
  - improving convergence through choice of weight
    - running of $\alpha_s(Q^2)$ $\Rightarrow$ convergence improved away from spacelike point
    - non-spectral weights, constructed to emphasize regions of improved convergence [e.g., w_{20}, $\hat{w}_{10}$, w_{10} (KM, Kambor PRD62(2000)093020)]
    - s_0-instability of physical output wrt s_0 as sign of premature truncation of slowly converging series
  - alternate flavor-breaking correlator combinations with suppressed $D = 2$ OPEs (more later)

$D > 4$ CONTRIBUTIONS

- $D = 4$ OPE fixed by $\langle m_s \bar{s} s \rangle$, $\langle m_\ell \bar{\ell} \ell \rangle$ (up to tiny $O(m_q^4)$ contributions) hence well constrained

- $D = 6$: VSA estimates only (larger deviations likely for flavor-breaking differences); $D > 6$: no reliable estimates

- For $w(s) \rightarrow w(y)$, $y = s/s_0$, integrated $D = 2k + 2$ contribution scales as $1/s_0^k$

- $\Rightarrow$ test assumptions/treatment of $D > 4$ contributions via stability of $|V_{us}|$ output wrt s_0

THE $w_{(0,0)}$ ANALYSIS (WITH SOME CAUTIONS)

- Experimental advantage: $s_0 = m_\tau^2$ FESR yields $|V_{us}|$ from branching fractions ONLY (i.e., without the more difficult $dR_{us;V+A}/ds$ distribution measurement)

- Gamiz et al 2007, updated for Belle 2007 $B[K\eta\nu_\tau]$, $B[K^*\eta\nu_\tau]$, BaBar 2008 $B[\bar{K}^0\pi^-\nu_\tau]$
  - Experimental input:

$$R_{ud;V+A} = 3.480(11),\ R_{us;V+A} = 0.1604(40)\ [K_{\mu 2}]$$
$$R_{ud;V+A} = 3.481(11),\ R_{us;V+A} = 0.1591(40)\ [B_K]$$

- The $\delta R_\tau \equiv \delta R^{w(0,0)}(m_\tau^2)$ assessment:
  * $\left[\delta R_\tau^{(0)}\right]_L = 0.1544(37)$ [0.1204 from K, π poles]
  * $\left[\delta R_\tau^{(0+1)}\right]_{OPE} = 0.0612(15)$

    [90% of uncertainty from m_s^2 $D = 2$ scale]
- results sensitive to small BR shifts

$$|V_{us}| = 0.2156(27)_{exp}(5)_{th} \quad K_{\mu 2}\ input$$
$$|V_{us}| = 0.2147(27)_{exp}(5)_{th} \quad B[K\nu_\tau]\ input$$

- dominant source of shift from Gamiz et al 0.2165 $K_{\mu 2}$ result is new $B[K\eta\nu_\tau]$, $B[K^*\eta\nu_\tau]$ values

- Some cautions re the nominal ± 0.0005 theory error
  - fixed $s_0 \Rightarrow$ no s_0-stability test (us covariances)
  - problematic convergence of integrated $D = 2$ (0+1) OPE series: $O(\bar{a}^4)$ assessments using Adler function (1^{st} line) or correlator (2^{nd} line)

$$\sim [1 + 0.285 + 0.097 - 0.054 - (0.224) + \cdots]$$
$$\sim [1 + 0.148 + 0.008 - 0.143 - (0.336) + \cdots]$$

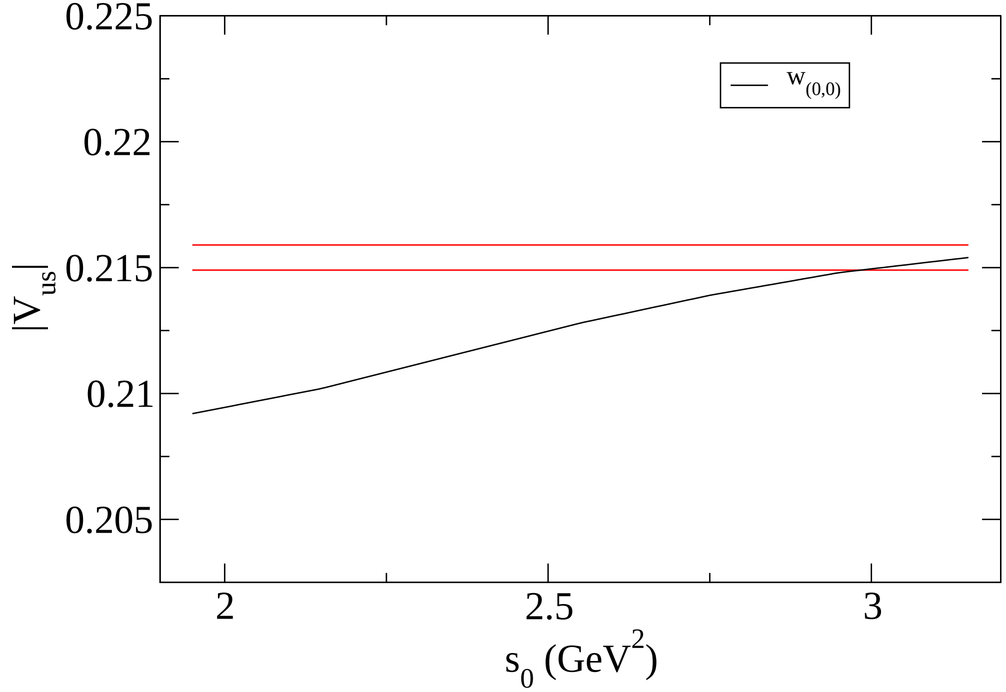
  * $O(a^3)$ Adler function $\rightarrow$ estimated $O(a^4)$ Adler function $D = 2$ assessment $\Rightarrow \delta|V_{us}| = -0.0003$
  * Contrast: $O(a^3)$ Adler function $\rightarrow$ estimated $O(a^4)$ correlator $D = 2$ assessment $\Rightarrow \delta|V_{us}| = -0.0008$

$\Rightarrow$ problematic convergence of integrated $D = 2$ $(0+1)$ series makes standard truncation error estimates insufficiently conservative

- alternate s_0-stability assessment
  - $s_0 \neq m_\tau^2$ results using mode-by-mode rescaled ALEPH 1999 us distribution
  - (thanks to S. Chen for us data and covariances)
  - significant s_0-instability for $w_{(0,0)}$ FESR [FIGURE]

instability $\Rightarrow$ either significant underestimate of $D = 2$ truncation error or $D = 6, 8$ contribution problems
$\Rightarrow$ hard to reliably quantify theory error for $w_{(0,0)}$ FESR

s_0-STABILITY, 2008 us DATA, $K_{\mu 2}$ INPUT



ALTERNATE CONVERGENCE/STABILITY STUDIES

- ALEPH us data, covariances; mode-by-mode rescaling for new BR's [Davier et al EPJC22 (2001) 31 strategy]

- significant central value shifts from recent B-factory results, especially $B[\bar{K}^0\pi^-\nu_\tau]$ (BELLE, arXiv:0706.2231, Banerjee ICHEP08), $B[K^-\pi^0\nu_\tau]$ (BABAR, arXiv:0707.2922), $B[K^-\pi^+\pi^-\nu_\tau]$ (BABAR, arXiv:0707.2981, Inami ICHEP08)

- TABLE for new 2008 WA's (Banerjee ICHEP08)

LEP+CLEO +BELLE+BABAR us $\mathcal{B}$ VALUES

Mode	$\mathcal{B}_{PDG06}$ (%)	$\mathcal{B}_{WA,2008}$ (%)
K^- [τ decay]	0.685(23)	0.692(12)
(Alt: [$K_{\mu 2}$])	(0.715(3))	(0.715(3))
$K^-\pi^0$	0.454(30)	0.426(16)
$\bar{K}^0\pi^-$	0.878(38)	0.835(22) ($S = 1.4$)
$K^-\pi^0\pi^0$	0.058(24)	[**]
$\bar{K}^0\pi^0\pi^-$	0.360(40)	[**]
$K^-\pi^-\pi^+$	0.330(50)	0.280(16) ($S = 1.9$)
$K^-\eta$	0.027(6)	0.016(2) ($S = 1.8$)
$(\bar{K}3\pi)^-$ (est'd)	0.074(30)	[†]
$K_1(1270) \to K^-\omega$	0.067(21)	[†]
$(\bar{K}4\pi)^-$ (est'd)	0.011(7)	[†]
$K^*\eta$	0.029(9)	0.012(4) ($S = 2.0$)
$K\phi$		0.004(0)
TOTAL	2.973(86) (3.003(83))	2.835(70) (2.858(70))

RESULTS/OBSERVATIONS

- Improved s_0 stability for non-spectral (improved $D = 2$ convergence) weights

- Window with very good s_0-stability for $\hat{w}_{10}$

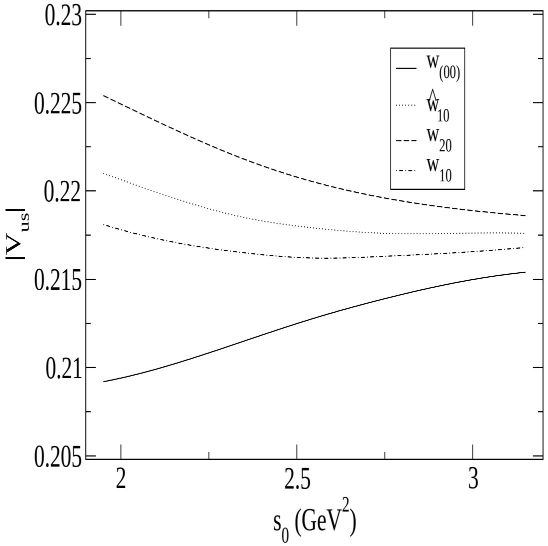
- Convergence toward $\hat{w}_{10}$ $|V_{us}|$ value for other weights (including $w_{(0,0)}$) as $s_0 \to m_\tau^2$

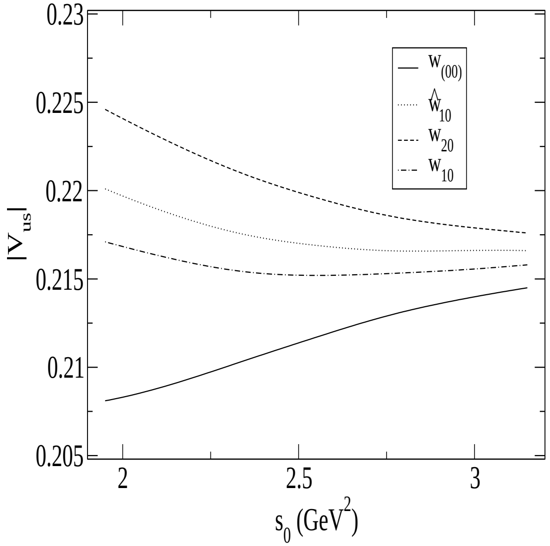
- No single- or multiple-mode rescalings studied produce improved $w_{(0,0)}$ $|V_{us}|$ s_0-stability

$|V_{us}|$ s_0 STABILITY

LHS: $K_{\mu 2}$ INPUT

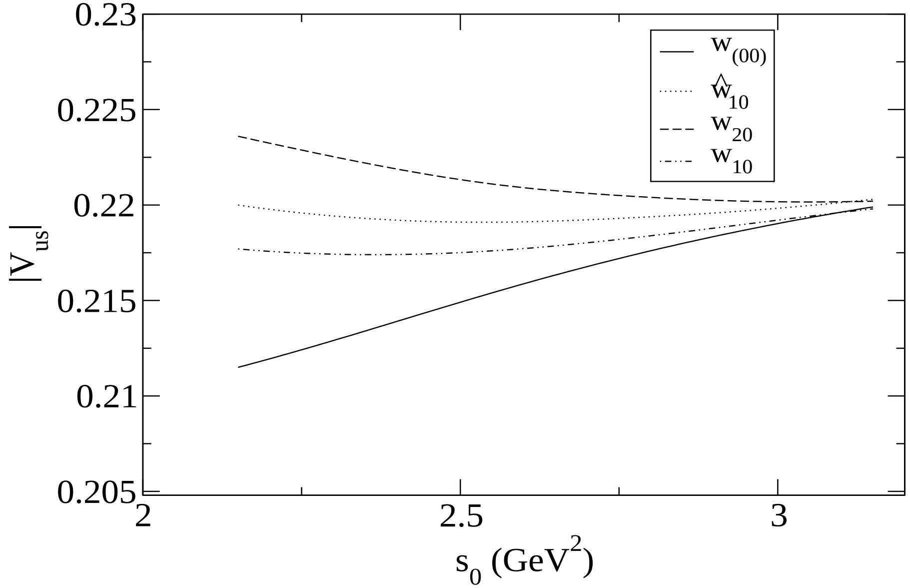
RHS: $B[K\nu_\tau]$ INPUT





- Full assessment of consistency of $s_0 = m_\tau^2$ values for different $w(y)$ awaits further data improvements

[e.g., results with $B[\tau \to \bar{K}^0\pi^-\pi^0\nu_\tau]$ scaled up 3σ



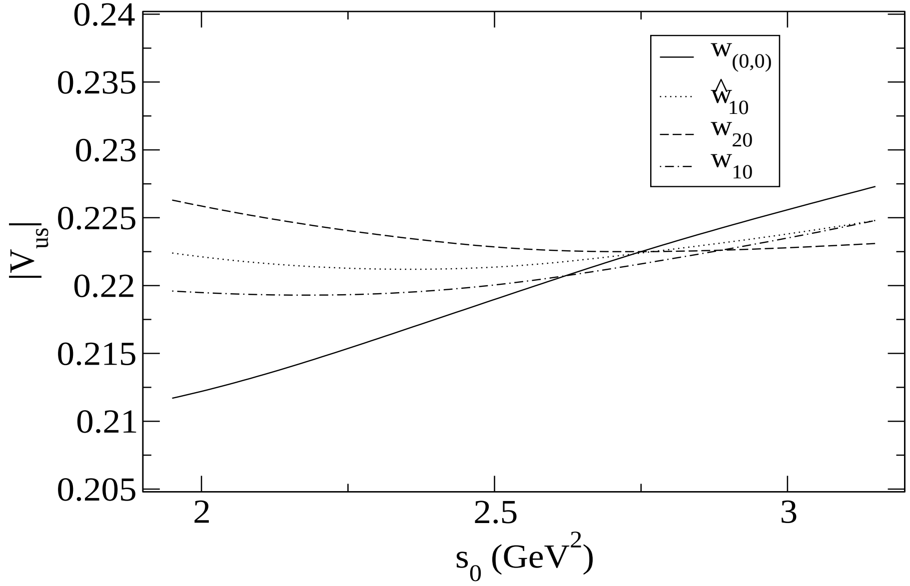
- $|V_{us}|$ from $s_0 = m_\tau^2$ single-weight fits, $K_{\mu 2}$ input ($D = 2$ overall m_s^2 scale dominates theory errors)

$$0.2177(32)_{exp}(19)_{th}\ (\hat{w}_{10})$$
$$0.2186(30)_{exp}(29)_{th}\ (w_{20})$$
$$0.2169(33)_{exp}(14)_{th}\ (w_{10})$$
$$0.2156(27)_{exp}(??)_{th}\ (w_{(0,0)})$$

- *$2-3\ \sigma$ from 3-family unitarity expectation,* 0.2255(10)*, most recent lattice-supplemented $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ results, AND ~ 0.0050 lower than 2006 results*

- Sizeable upward shifts for un(re)measured us modes needed for agreement with 3-fmaily unitarity [FIGURE]

$|V_{us}|$ results for us spectrum with all modes not yet re-measured by BaBar and/or Belle scaled up by 3σ



- Re the $\sim$ 0.0050 decrease since 2006: shifts in B_{us} appear small (sub-0.1%) but

  - shift in $\mathcal{B}[K^-\pi^0\nu_\tau]+\mathcal{B}[\bar{K}^0\pi^-\nu_\tau]$ ($-$0.071%) is $\sim$ 2.6% of total us branching fraction $\leftrightarrow$ $\sim$ 1.3% ($\sim$ 0.0029) reduction in V_{us}

  - shift in $\mathcal{B}[K^-\pi^-\pi^+\nu_\tau]$ ($-$0.050%) is $\sim$ 1.8% of total us branching fraction $\leftrightarrow$ $\sim$ 0.9% ($\sim$ 0.0020) reduction in V_{us}

  $\Rightarrow$ $\sim$ 0.0050 reduction in V_{us} c.f. 2006 analysis values

- LESSON: will need all strange modes (some no doubt new) with BF's down to $\sim$ a few 10^{-5} level

ALTERNATE FESR CHOICES

- Slow convergence of the integrated $D = 2$ OPE series for $\Delta\Pi$ due to slow convergence at the correlator level (for scales kinematically accessible in τ decay)

- Suggests trying alternate flavor-breaking combinations with suppressed $D = 2$ OPE contributions, e.g.,

$$\Delta\Pi^{EM,\tau} \equiv 3\Pi_{EM} - \left[\frac{4}{3}\Pi_{ud;V} + \frac{1}{3}\Pi_{us:V+A}\right]$$

(normalization chosen to match cancelling $D = 0$ scales with those of $\Delta\Pi$)

- $D = 2$ suppression choice also suppresses $D = 4$

  - $D = 2$

$$\left[\Delta\Pi(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[1 + \frac{7}{3}\bar{a} + 19.933\bar{a}^2 + 208.75\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2)\right]_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[0 + \frac{1}{9}\bar{a} + 1.4613\bar{a}^2 + 14.981\bar{a}^3 + \cdots\right]$$

  - $D = 4$

$$\left[\Delta\Pi(Q^2)\right]_{D=4} = \frac{\langle m_s\bar{s}s\rangle - \langle m_\ell\bar{\ell}\ell\rangle}{Q^4}\left[-2 - 2\bar{a} - \frac{26}{3}\bar{a}^2\right]$$

$$\left[\Delta\Pi^{EM,\tau}(Q^2)\right]_{D=4} = \frac{\langle m_s\bar{s}s\rangle - \langle m_\ell\bar{\ell}\ell\rangle}{Q^4}\left[0 + \frac{8}{9}\bar{a} + \frac{59}{9}\bar{a}^2\right]$$

- Strong suppression of $D = 2,4$ contributions $\Rightarrow$ $w(y)$ usable even without improved $D = 2$ convergence, hence e.g. $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$
  - single integrated $D > 4$ contribution ($D = 2N + 2$) (up to $O(\alpha_s^2)$ corrections)
  - $D = 2N + 2$ suppressed by relevant w_N coefficient, $1/(N-1)$
  - $1/s_0^N$ dependence provides handle on integrated $D = 2N + 2$ contributions

- NOTE: $D > 4$ (typically NOT suppressed at correlator level) potentially more relevant due to 3-fold reduction of non-OPE denominator term ($3R^w_{EM} - \frac{4}{3}R^w_{ud;V}$ vs. $R^w_{ud;V+A}$ for the pure τ case)

RESULTS OF THE MIXED τ-EM FESR ANALYSIS

- Presently complicated by inconsistency of $I = 1$ EM and τ results, even after IB corrections

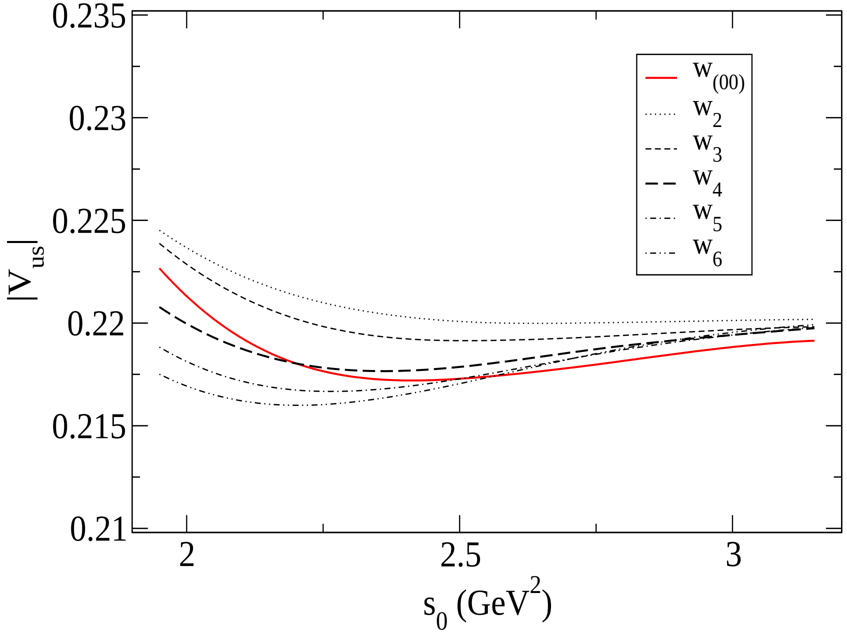
- EM $\pi\pi$ and 4π spectral contributions "corrected" using τ data (choice predicated by comparison to lattice α_s)

- Only s_0-instability should be from $D > 4$ terms

- Observe good consistency, stability [FIGURE]

- Significantly improved stability for $w_{(0,0)}$ (compatible with instability for τ due to slow $D = 2$ convergence)

V_{us} vs. s_0 from the mixed τ decay-EM FESRs



- Result for best stability case (w_2) is

$$|V_{us}| = 0.2202(63)$$

with 0.0042 of the error from the us spectral integral

- Obviously preliminary/a demonstration of principle at present due to EM-τ discrepancy, but a potentially useful method in the longer run

- Other flavor-breaking combinations with simultaneously suppressed $D = 2, 4$ OPE correlator contributions also exist (additional consistency checks hence possible)

PROSPECTS/CONCLUSIONS

- Many us BF errors already much reduced, others soon to be much reduced by BaBar, Belle

- ingredients for full remeasurement of actual us spectral distribution in place and work in progress

- Some obvious targets for near term BaBar, Belle work ($K_S\pi^-\pi^0$, $K^-\pi^0\pi^0$, $K3\pi$, $K\omega$, $K4\pi$)

- Desirability of BaBar, Belle cross-checks, detailed studies of higher multiplicity modes ($\bar{K}3\pi$, $\bar{K}4\pi, \cdots$)

- Need all strange modes with B to few-10^{-5} level
  - $B[K^-\phi]$ at few $\times 10^{-5}$ already reported
  - missing modes: higher multiplicity, higher s region
  - total $B_{us} \sim 3\%$ $\Rightarrow$ neglected 10^{-4} mode lowers $|V_{us}|$ by ~ 0.0004 for $w^{(0,0)}$, somewhat less for $w(s)$ with stronger high-s suppression

- Poor convergence of $D = 2$ $(0,0)$ spectral weight OPE series implies need re-measured spectral *distribution* NOT just improved branching fractions (unfortunately)

- ud data also relevant [e.g. reduced $\pi\pi$ contribution to $\rho^{(1)}_{ud;V}$ implied by e^+e^- data would raise $|V_{us}|$ by ~ 0.0018)] (relation to $(g-2)_\mu$ question)

- Additional non-spectral weights possibile, but require improved us distributions for exploration, improvement

- s_0-stability tests CRUCIAL given slow convergence of the $D = 2$ $J = 0 + 1$ OPE series

- *Experimentalists: public accessibility of covariance matrices crucial for $s_0 \neq m_\tau^2$, s_0-stability tests, non-spectral weight analyses, new weight explorations*

- Two possible scenarios for the current $2-3\sigma$ discrepancy with 3-family unitarity
  - Continuing discrepancy with unitarity expectations (obviously the most interesting possibility)
  - New mode results, shifts for remaining modes restore agreement, in which case
    - $|V_{us}|$ to sub-0.0010 accuracy from m_s to ± 5 MeV, better than 1/5 us data error reduction
    - $|V_{us}|$ uncertainties comparable to AND *completely independent of* those for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice chiral extrapolation), hence further improvement by averaging

SUPPLEMENTARY PAGES

- Details on the handling of potential $D > 6$ OPE contributions

- Further evidence for instability of $(0,0)$ spectral weight analyis, improved stability for non-spectral weights

HIGHER D OPE CONTRIBUTIONS

- rough estimates for $D = 6$ condensates, $D > 6$ combinations unknown, usually assumed negligible

- $w(y) = \sum_m c_m y^m$, $y = s/s_0$ $\Rightarrow$ integrated $D = 2k + 2$ OPE $\propto c_k/s_0^k$ (up to logs) $\Rightarrow$ avoid large c_k, $k \geq 2$

- neglect of non-negligible higher D terms $\Rightarrow$ s_0-instability of output $\Rightarrow$ *need to study output as function of* s_0

- NOTE: growth of coefficients in $(k,0)$ spectral weights

$$
\begin{aligned}
w^{(0,0)}(y) &= 1-3y^2+2y^3\\
w^{(1,0)}(y) &= 1-y-3y^2+5y^3-2y^4\\
w^{(2,0)}(y) &= 1-2y-2y^2+8y^3-7y^4+2y^5\\
w^{(3,0)}(y) &= 1-3y+10y^3-15y^4+9y^5-2y^6\\
w^{(4,0)}(y) &= 1-4y+3y^2+10y^3-25y^4+24y^5\\
&\quad -11y^6+2y^7
\end{aligned}
$$

- contrast 4 *non-spectral weights* used in literature (also normalized to 1 at $y=0$): w_{20}, $\widehat{w}_{10}$, w_{10}, w_8, with largest $k \geq 2$ coefficients $c_4 = 2.087$ (w_{20}), $c_5 = 1.206$ $(\widehat{w}_{10})$, $c_5 = 2$ (w_{10}), $c_5 = 1.182$ (w_8)

Further Evidence of Instability for the $(0,0)$ Spectral Weight, Improved Stability for Non-Spectral Weights

$O(a^N)$-truncated $D = 2$ correlator/Adler function difference as alternate estimate of truncation uncertainty

- $r_k^w(s_0)$: $O(a^k)$ (correlator-Adler)/correlator ratio

Weight	$r_1^w(m_\tau^2)$	$r_2^w(m_\tau^2)$	$r_3^w(m_\tau^2)$	$r_4^w(m_\tau^2)$
$w^{(0,0)}_{J=0+1}$	-0.01	0.06	0.20	0.67
$\hat{w}_{10}$	-0.11	-0.07	-0.05	-0.03
w_{20}	-0.11	-0.08	-0.05	-0.03
w_{10}	-0.10	-0.06	-0.03	-0.01