

Model-Independent Analysis of $\tau \rightarrow \ell\ell'$ Decays

Sascha Turczyk

in collaboration with

Benjamin Dassingier Thorsten Feldmann Thomas Mannel

based on [[JHEP 0710 \(2007\) 039](#)]

Theoretische Physik 1
Universität Siegen

Tau 2008 Workshop, Budker Institute Novosibirsk
September 23, 2008

Content

- 1 Introduction
 - Motivation
 - Idea of the Analysis

- 2 Theoretical Basics
 - Operator Analysis
 - Decay Modes
 - Conventions

- 3 Results
 - $\tau^- \rightarrow \mu^- \mu^- \mu^+$
 - $\tau^- \rightarrow e^- \mu^- \mu^+$
 - Comment on Minimal Flavour Violation Scenarios

Motivation

Why New Physics?

- First evidence of New physics (NP): Neutrino oscillation
- Incorporating neutrino oscillation into SM
- ⇒ Lepton flavour violating (LFV) decays, but practically unobservable in the SM
- New physics at the TeV scale
- ⇒ Potentially sizeable LFV decays

How to Identify Beyond SM Effects

- Reliable experimental bounds on LFV decay modes
- ⇒ Requires knowledge on phase space dependence
- Compare particular NP models (SUSY, Little Higgs, ...)
- Or: Model-independent approach (this work)

Motivation

Why New Physics?

- First evidence of New physics (NP): Neutrino oscillation
- Incorporating neutrino oscillation into SM
- ⇒ Lepton flavour violating (LFV) decays, but practically unobservable in the SM
- New physics at the TeV scale
- ⇒ Potentially sizeable LFV decays

How to Identify Beyond SM Effects

- Reliable experimental bounds on LFV decay modes
- ⇒ Requires knowledge on phase space dependence
- Compare particular NP models (SUSY, Little Higgs, ...)
- Or: Model-independent approach (this work)

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...
- ⇒ Depending on NP model: Dominance of particular operators
- ⇒ Consistent exp. constraints on NP for a given scenario
- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC
- ⇒ Here: Concentrate on $\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-$ and $\tau^\pm \rightarrow e^\pm \mu^+ \mu^-$

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...
- ⇒ Depending on NP model: Dominance of particular operators
- ⇒ Consistent exp. constraints on NP for a given scenario
- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC
- ⇒ Here: Concentrate on $\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-$ and $\tau^\pm \rightarrow e^\pm \mu^+ \mu^-$

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...
- ⇒ Depending on NP model: Dominance of particular operators
- ⇒ Consistent exp. constraints on NP for a given scenario
- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC
- ⇒ Here: Concentrate on $\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-$ and $\tau^\pm \rightarrow e^\pm \mu^+ \mu^-$

General Operators

Structure

- Respect Gauge Symmetry of SM
- Group leptons into (incomplete) Doublets ν : Higgs VEV

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ \ell_R \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & \nu + h_0 - i\chi_0 \end{pmatrix}$$

dim=6 leptonic:

$$O_1 = (\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$$

$$O_2 = (\bar{L}\tau^a\gamma_\mu L)(\bar{L}\tau^a\gamma^\mu L)$$

$$O_3 = (\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R)$$

$$O_4 = (\bar{R}\gamma_\mu R)(\bar{L}\gamma^\mu L)$$

dim=6 radiative:

$$R_1 = g'(\bar{L}H\sigma_{\mu\nu}R)B^{\mu\nu}$$

$$R_2 = g(\bar{L}\tau^a H\sigma_{\mu\nu}R)W^{\mu\nu,a}$$

Ingredients

- $B^{\mu\nu}$: $U(1)_Y$ and $W^{\mu\nu,a}$: $SU(2)_L$ gauge field
- Only tree-level contribution for simplicity

► General Case

General Operators

Structure

- Respect Gauge Symmetry of SM
- Group leptons into (incomplete) Doublets ν : Higgs VEV

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ \ell_R \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & \nu + h_0 - i\chi_0 \end{pmatrix}$$

dim=6 leptonic:

$$O_1 = (\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$$

$$O_2 = (\bar{L}\tau^a\gamma_\mu L)(\bar{L}\tau^a\gamma^\mu L)$$

$$O_3 = (\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R)$$

$$O_4 = (\bar{R}\gamma_\mu R)(\bar{L}\gamma^\mu L)$$

dim=6 radiative:

$$R_1 = g'(\bar{L}H\sigma_{\mu\nu}R)B^{\mu\nu}$$

$$R_2 = g(\bar{L}\tau^a H\sigma_{\mu\nu}R)W^{\mu\nu,a}$$

Ingredients

- $B^{\mu\nu}$: $U(1)_Y$ and $W^{\mu\nu,a}$: $SU(2)_L$ gauge field
- Only tree-level contribution for simplicity

► General Case

Relevant Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}, \quad H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar{\ell} \ell | H_{\text{eff}}^{\text{rad}} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} \langle \ell' \bar{\ell} \ell | \left(\bar{\ell}_h (-i\sigma_{\mu\nu}) \tau_s \right) (\bar{\ell}' \gamma^\mu \ell) | \tau \rangle, \quad (s, h) = \begin{pmatrix} L, R \\ R, L \end{pmatrix}$$

Relevant Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}, \quad H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar{\ell} \ell | H_{\text{eff}}^{\text{rad}} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} \langle \ell' \bar{\ell} \ell | (\bar{\ell}_h (-i\sigma_{\mu\nu}) \tau_s) (\bar{\ell} \gamma^\mu \ell) | \tau \rangle, \quad (s, h) = \begin{pmatrix} L, R \\ R, L \end{pmatrix}$$

Relevant Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}, \quad H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar{\ell} \ell | H_{\text{eff}}^{\text{rad}} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} \langle \ell' \bar{\ell} \ell | \left(\bar{\ell}'_h (-i\sigma_{\mu\nu}) \tau_s \right) \left(\bar{\ell} \gamma^\mu \ell \right) | \tau \rangle, \quad (s, h) = \begin{pmatrix} L, R \\ R, L \end{pmatrix}$$

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\text{rad}}^{(L_i R^j)}$, $g_{\text{rad}}^{(R_i L^j)}$

The Different Decay Modes

$$\tau^- \rightarrow e^- e^- e^+$$

$$\tau^- \rightarrow \mu^- \mu^- \mu^+$$

$$\tau^- \rightarrow e^- e^- \mu^+$$

$$\tau^- \rightarrow \mu^- \mu^- e^+$$

$$\tau^- \rightarrow \mu^- e^- e^+$$

$$\tau^- \rightarrow e^- \mu^- \mu^+$$

- Take into account Pauli principle for two identical particles
- Concentrate on red marked: Contribution from $H_{\text{eff}}^{\text{rad}}$

$$\tau^- \rightarrow l'^- \gamma^* \rightarrow l'^- (l^+ l^-)$$

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\text{rad}}^{(L_i R^j)}$, $g_{\text{rad}}^{(R_i L^j)}$

The Different Decay Modes

$$\tau^- \rightarrow e^- e^- e^+$$

$$\tau^- \rightarrow \mu^- \mu^- \mu^+$$

$$\tau^- \rightarrow e^- e^- \mu^+$$

$$\tau^- \rightarrow \mu^- \mu^- e^+$$

$$\tau^- \rightarrow \mu^- e^- e^+$$

$$\tau^- \rightarrow e^- \mu^- \mu^+$$

- Take into account Pauli principle for two identical particles
- Concentrate on red marked: Contribution from $H_{\text{eff}}^{\text{rad}}$

$$\tau^- \rightarrow \ell'^- \gamma^* \rightarrow \ell'^- (\ell^+ \ell^-)$$

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\text{rad}}^{(L_i R^j)}$, $g_{\text{rad}}^{(R_i L^j)}$

The Different Decay Modes

$$\tau^- \rightarrow e^- e^- e^+$$

$$\tau^- \rightarrow \mu^- \mu^- \mu^+$$

$$\tau^- \rightarrow e^- e^- \mu^+$$

$$\tau^- \rightarrow \mu^- \mu^- e^+$$

$$\tau^- \rightarrow \mu^- e^- e^+$$

$$\tau^- \rightarrow e^- \mu^- \mu^+$$

- Take into account Pauli principle for two identical particles
- Concentrate on red marked: Contribution from $H_{\text{eff}}^{\text{rad}}$

$$\tau^- \rightarrow \ell'^- \gamma^* \rightarrow \ell'^- (\ell^+ \ell^-)$$

Conventions

Kinematical Variables

- Neglect electron mass
- Decay distribution (Dalitz distribution) in

$$m_{--}^2 \equiv m_{12}^2 = (p'_{e^-, \mu^-} + p_{\mu^-})^2$$

$$m_{+-}^2 \equiv m_{23}^2 = (p_{\mu^-} + p_{\mu^+})^2$$

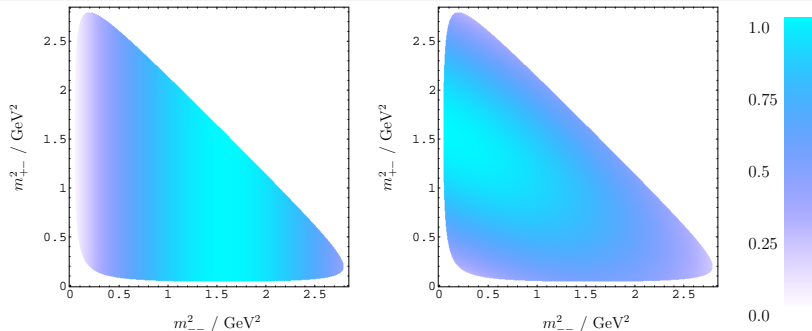
- Third combination is given by

$$m_{13}^2 = m_{\tau}^2 + 2m_{\mu}^2 + m_{e^-, \mu^-}^2 - m_{--}^2 - m_{+-}^2$$

Decay Distributions

- Consider Dalitz distributions in m_{--}^2 and m_{+-}^2
- Neglect interference terms with different helicities (suppression due to small lepton masses)

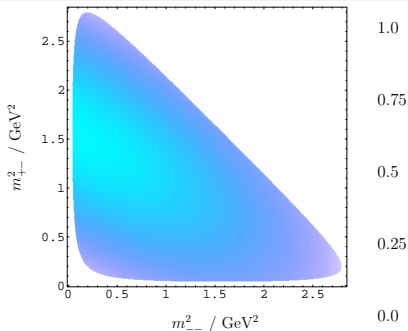
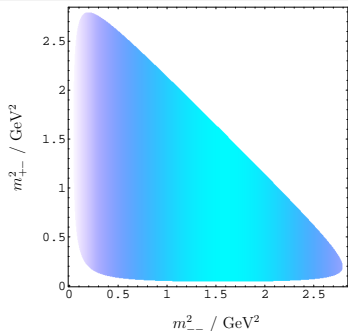
Vector Current



Decay Distribution for Left Figure: $\propto |g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}|^2$

$$\frac{d^2 \Gamma_V^{(LL)(LL)}}{dm_{23}^2 dm_{12}^2} = \frac{|g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4} \frac{(m_\tau^2 - m_\mu^2)^2 - (2m_{12}^2 - m_\tau^2 - 3m_\mu^2)^2}{256 \pi^3 m_\tau^3}$$

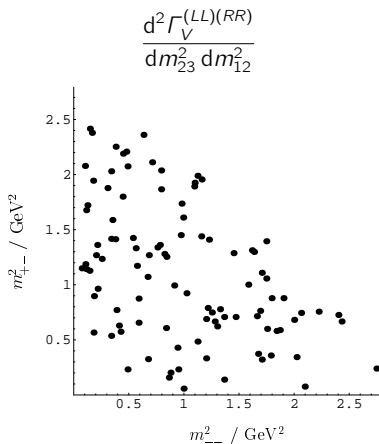
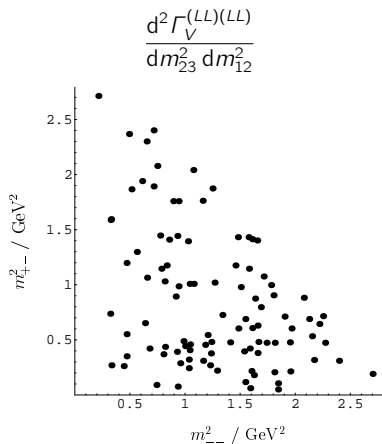
Vector Current



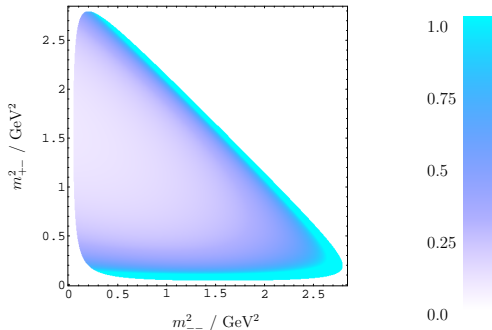
Decay Distribution for Right Figure: $\propto |g_V^{(L_\mu L^T)(R_\mu R^\mu)}|^2$

$$\frac{d^2 \Gamma_V^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} = \frac{|g_V^{(L_\mu L^T)(R_\mu R^\mu)}|^2}{\Lambda^4} \left[\frac{(m_\tau^2 - m_\mu^2)^2 - 4m_\mu^2 (m_\tau^2 + m_\mu^2 - m_{12}^2)}{512 \pi^3 m_\tau^3} - \frac{(2m_{13}^2 - m_\tau^2 - 3m_\mu^2)^2 + (2m_{23}^2 - m_\tau^2 - 3m_\mu^2)^2}{1024 \pi^3 m_\tau^3} \right]$$

Simulation for 100 Events: Vector Current



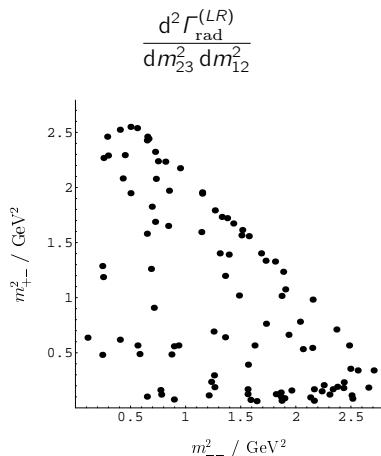
Radiative Transition



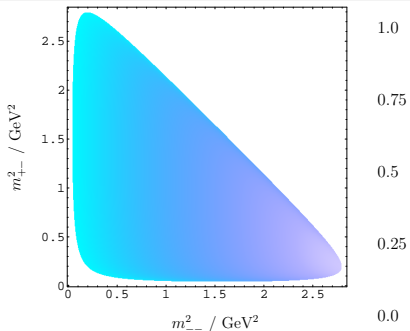
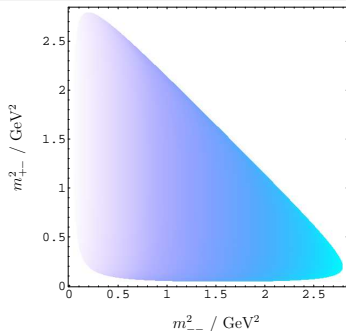
Decay Distribution for Radiative Mediated Decay

$$\frac{d^2 \Gamma_{\text{rad}}^{(LR)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}}^2 \frac{|g_{\text{rad}}^{(L\mu R\tau)}|^2 v^2}{\Lambda^4} \left[\frac{m_\mu^2 (m_\tau^2 - m_\mu^2)^2}{128 \pi^3 m_\tau^3} \left(\frac{1}{m_{13}^4} + \frac{1}{m_{23}^4} \right) + \frac{2m_{12}^2 - 3m_\mu^2}{128 \pi^3 m_\tau^3} \right. \\ \left. + \frac{m_\mu^2 (m_\tau^4 - 3m_\tau^2 m_\mu^2 + 2m_\mu^4)}{128 \pi^3 m_{13}^2 m_{23}^2 m_\tau^3} + \frac{(m_{13}^2 + m_{23}^2)(m_{12}^4 + m_{13}^4 + m_{23}^4 - 6m_\mu^2(m_\mu^2 + m_\tau^2))}{256 \pi^3 m_{13}^2 m_{23}^2 m_\tau^3} \right]$$

Simulation for 100 Events: Radiative Transition



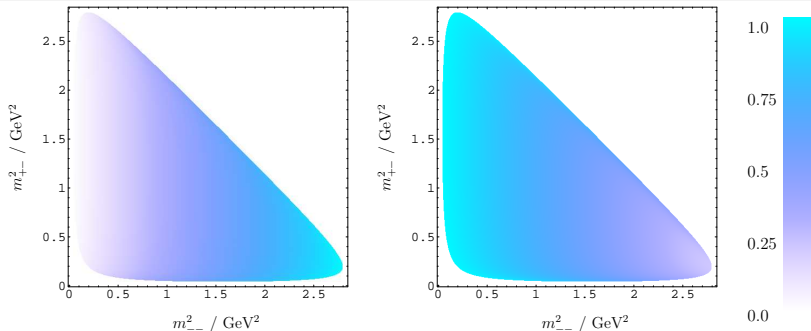
Interference Terms



Decay Distribution for Left Figure: $\propto \text{Re}[g_V^{(L_\mu L^T)(L_\mu L^\mu)} g_{\text{rad}}^{*(L_\mu R^T)}]$

$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(LL)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}} \frac{2 v \text{Re}[g_V^{(L_\mu L^T)(L_\mu L^\mu)} g_{\text{rad}}^{*(L_\mu R^T)}]}{\Lambda^4} \times \left[\frac{m_{12}^2 - 3m_\mu^2}{64 \pi^3 m_\tau^2} + \frac{m_\mu^2 (m_\tau^2 - m_\mu^2) (m_{13}^2 + m_{23}^2)}{128 \pi^3 m_\tau^2 m_{13}^2 m_{23}^2} \right]$$

Interference Terms

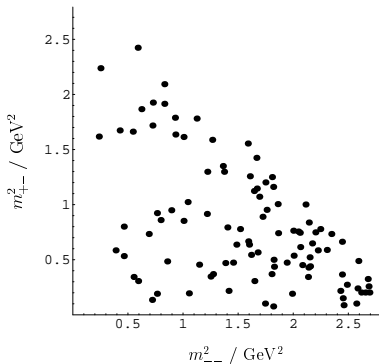


Decay Distribution for Right Figure: $\propto \text{Re}[g_V^{(L_\mu L^T)(R_\mu R^\mu)} g_{\text{rad}}^{*(L_\mu R^T)}]$

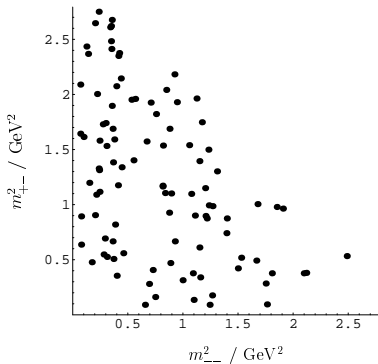
$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}} \frac{2 v \text{Re}[g_V^{(L_\mu L^T)(R_\mu R^\mu)} g_{\text{rad}}^{*(L_\mu R^T)}]}{\Lambda^4} \times \left[\frac{m_\tau^2 - m_{12}^2 - 3m_\mu^2}{256 \pi^3 m_\tau^2} + \frac{m_\mu^2 (m_\tau^2 - m_\mu^2) (m_{13}^2 + m_{23}^2)}{256 \pi^3 m_\tau^2 m_{13}^2 m_{23}^2} \right]$$

Simulation for 100 Events: Interference Terms

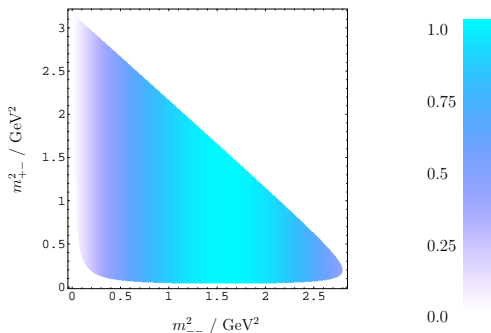
$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(LL)}}{dm_{23}^2 dm_{12}^2}$$



$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2}$$



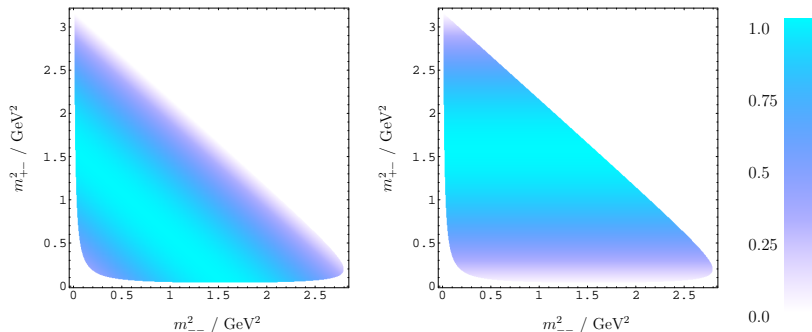
Vector Current Purely Left Handed



Decay Distribution

$$\frac{d^2 \Gamma_V^{(LL)(LL)}}{dm_{23}^2 dm_{12}^2} = \frac{|g_V^{(L_e L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4} \frac{m_\tau^4 - (2m_{12}^2 - m_\tau^2 - 2m_\mu^2)^2}{512 \pi^3 m_\tau^3}$$

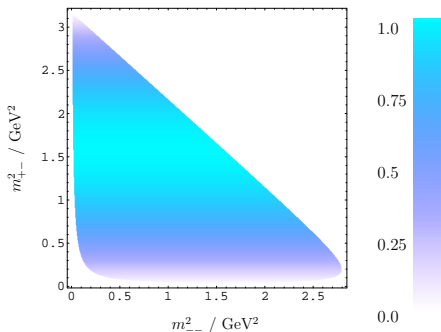
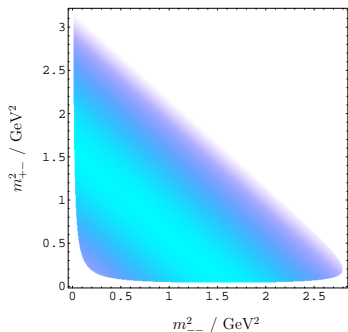
Vector Current



Decay Distribution for Left Figure: Left Handed Electron

$$\left. \frac{d^2 \Gamma_V^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} \right|_{e_L} = \frac{|g_V^{(L_e L^T)(R_\mu R^\mu)}|^2}{\Lambda^4} \frac{m_\tau^4 - (2m_{13}^2 - m_\tau^2 - 2m_\mu^2)^2}{512 \pi^3 m_\tau^3}$$

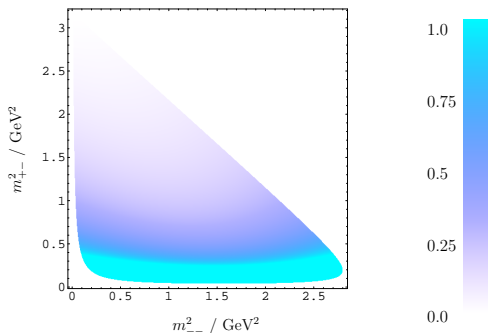
Vector Current



Decay Distribution for Right Figure: Right Handed Electron

$$\left. \frac{d^2 \Gamma_V^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} \right|_{e_R} = \frac{|g_V^{(L_\mu L^T)(R_e R^\mu)}|^2}{\Lambda^4} \frac{(m_\tau^2 - 2m_\mu^2)^2 - (2m_{23}^2 - m_\tau^2 - 2m_\mu^2)^2}{512 \pi^3 m_\tau^3}$$

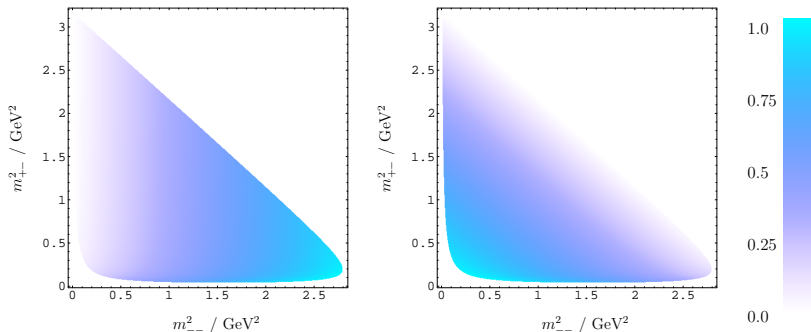
Radiative Transition



Decay Distribution for Radiative Mediated Decay

$$\frac{d^2\Gamma_{\text{rad}}^{(LR)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}}^2 \frac{|g_{\text{rad}}^{(L_e R^T)}|^2 v^2}{\Lambda^4} \left[\frac{m_\mu^2 (m_{23}^2 - m_\tau^2)^2}{64\pi^3 m_\tau^3 m_{23}^4} + \frac{m_{12}^4 + m_{13}^4 - 2m_\mu^4}{128\pi^3 m_\tau^3 m_{23}^2} + \frac{m_\tau^2 - m_{23}^2}{128\pi^3 m_\tau^3} \right]$$

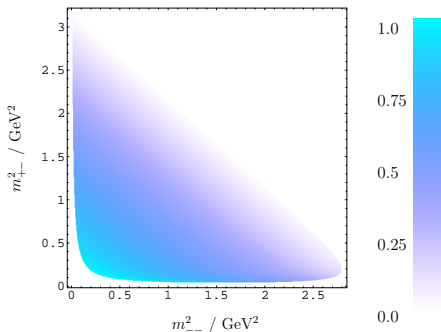
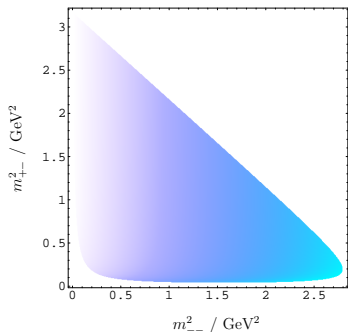
Interference Terms



Decay Distribution for Left Figure: $\propto \text{Re}[g_V^{(L_e L^T)(L_\mu L^\mu)} g_{\text{rad}}^{*(L_e R^T)}]$

$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(LL)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}} \frac{2 v \text{Re}[g_V^{(L_e L^T)(L_\mu L^\mu)} g_{\text{rad}}^{*(L_e R^T)}]}{\Lambda^4} \left[\frac{m_{12}^2 - 2m_\mu^2}{128\pi^3 m_\tau^2} + \frac{m_\mu^2}{128\pi^3 m_{23}^2} \right]$$

Interference Terms



Decay Distribution for Right Figure: $\propto \text{Re}[g_V^{(L_e L^T)(R_\mu R^\mu)} g_{\text{rad}}^{*(L_e R^T)}]$

$$\frac{d^2 \Gamma_{\text{mix}}^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} = \alpha_{\text{em}} \frac{2 v \text{Re}[g_V^{(L_e L^T)(R_\mu R^\mu)} g_{\text{rad}}^{*(L_e R^T)}]}{\Lambda^4} \left[\frac{m_{13}^2 - 2m_\mu^2}{128\pi^3 m_\tau^2} + \frac{m_\mu^2}{128\pi^3 m_{23}^2} \right]$$

Comment on Minimal Flavour Violation

$SU(3)_L \times SU(3)_{E_R}$ Flavour Symmetry Breaking

- 2 spurions in minimal extension of SM: $\lambda \sim (\bar{3}, 3)$ and $g_\nu \sim (\bar{6}, 1)$

$$\lambda = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau)$$

$$g_\nu = \frac{\Lambda_{\text{LN}}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2}) U^\dagger$$

\Rightarrow Parametrised by lepton masses and PMNS matrix U

Spurions in 2-Lepton and 4-Lepton Operators

- Right handed field require λ insertion $\Rightarrow m_\ell/v$ suppression
- Tensor reduction for $L^i L^j L_k^* L_j^*$: $\bar{6} \times 6 = 1 + 8 + 27$
with octet Δ_{ij}^k and 27plet G_{ij}^{kl}
- 2-lepton operators only involve octet Δ_{ij}^k

Combinations for Relevant Couplings

$$g_V^{(L_k L^i)(L_l L^j)} \rightarrow 2c_1 \Delta_i^k \delta_j^l + c_2 G_{ij}^{kl}$$

$$g_V^{(L_k L^i)(R_l R^j)} \rightarrow c_3 \Delta_i^k \delta_j^l$$

$$g_{\text{rad}}^{(L_k R^i)} \rightarrow c_4 \Delta_i^k \lambda_i$$

Comment on Minimal Flavour Violation

$SU(3)_L \times SU(3)_{E_R}$ Flavour Symmetry Breaking

- 2 spurions in minimal extension of SM: $\lambda \sim (\bar{3}, 3)$ and $g_\nu \sim (\bar{6}, 1)$

$$\lambda = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau) \quad g_\nu = \frac{\Lambda_{\text{LN}}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2}) U^\dagger$$

⇒ Parametrised by lepton masses and PMNS matrix U

Spurions in 2-Lepton and 4-Lepton Operators

- Right handed field require λ insertion ⇒ m_ℓ/v suppression
- Tensor reduction for $L^i L^j L_k^* L_j^*$: $\bar{6} \times 6 = 1 + 8 + 27$
with octet Δ_i^k and 27plet G_{ij}^{kl}
- 2-lepton operators only involve octet Δ_i^k

Combinations for Relevant Couplings

$$g_V^{(L_k L^i)(L_l L^j)} \rightarrow 2c_1 \Delta_i^k \delta_j^l + c_2 G_{ij}^{kl} \quad g_V^{(L_k L^i)(R_l R^j)} \rightarrow c_3 \Delta_i^k \delta_j^l \quad g_{\text{rad}}^{(L_k R^i)} \rightarrow c_4 \Delta_i^k \lambda_l$$

Comment on Minimal Flavour Violation

$SU(3)_L \times SU(3)_{E_R}$ Flavour Symmetry Breaking

- 2 spurions in minimal extension of SM: $\lambda \sim (\bar{3}, 3)$ and $g_\nu \sim (\bar{6}, 1)$

$$\lambda = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau) \quad g_\nu = \frac{\Lambda_{LN}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2}) U^\dagger$$

⇒ Parametrised by lepton masses and PMNS matrix U

Spurions in 2-Lepton and 4-Lepton Operators

- Right handed field require λ insertion ⇒ m_ℓ/v suppression
- Tensor reduction for $L^i L^j L_k^* L_j^*$: $\bar{6} \times 6 = 1 + 8 + 27$
with octet Δ_{ij}^k and 27plet G_{ij}^{kl}
- 2-lepton operators only involve octet Δ_{ij}^k

Combinations for Relevant Couplings

$$g_V^{(L_k L^i)(L_l L^j)} \rightarrow 2c_1 \Delta_i^k \delta_j^l + c_2 G_{ij}^{kl} \quad g_V^{(L_k L^i)(R_l R^j)} \rightarrow c_3 \Delta_i^k \delta_j^l \quad g_{\text{rad}}^{(L_k R^i)} \rightarrow c_4 \Delta_i^k \lambda_i$$

Comment on Minimal Flavour Violation

Results for Relevant Combinations (limit of small mixing angle)

- Octet $\Delta_{\tau}^{\mu} = \mathcal{O}\left(\frac{\Lambda_{LN}^2}{v^4} \Delta m_{\text{atm}}^2\right)$ $\Delta_{\tau}^e = \mathcal{O}\left(\frac{\Lambda_{LN}^2}{v^4} \Delta m_{\text{atm}}^2 \sin \theta_{13}\right)$

- 27plet in the limit of vanishing Majorana phases
 - In the case of normal neutrino hierarchy

$$G_{\tau\mu}^{\mu\mu} \simeq \frac{\Lambda_{LN}^2}{v^4} \frac{\Delta m_{\text{atm}}^2}{20}$$

$$G_{\tau\mu}^{e\mu} \simeq \frac{\Lambda_{LN}^2}{v^4} \frac{\sqrt{\Delta m_{\text{atm}}^2}}{2} \left(\frac{3 \cos \delta - 7i \sin \delta}{5} \sin \theta_{13} \sqrt{\Delta m_{\text{atm}}^2} \right. \\ \left. + \sin 2\theta_{12} \frac{\Delta m_{\text{sol}}^2}{4m_{\nu_{1,2}}} \right)$$

- In the case of inverted neutrino hierarchy

$$G_{\tau\mu}^{\mu\mu} \simeq -\frac{\Lambda_{LN}^2}{2v^4} \frac{\Delta m_{\text{atm}}^2}{20} \quad G_{\tau\mu}^{e\mu} \simeq \frac{\Lambda_{LN}^2}{v^4} \frac{7\Delta m_{\text{atm}}^2}{10\sqrt{2}} e^{i\delta} \sin \theta_{13}$$

Summary

Summary

- LFV provides important test of SM against NP
- Here: Made a model-independent analysis of $\tau \rightarrow lll$ decays
- Classified different operators in EFT
- Calculated Dalitz distributions for individual structures including interference terms

Conclusion

- Dalitz distributions provide a tool to distinguish NP models complementary to e.g. $\Gamma(\tau \rightarrow 3l)/\Gamma(\tau \rightarrow l\gamma)$
- $\mathcal{O}(100)$ measured events may be sufficient to distinguish effects from radiative and four-lepton-operators
- Experimental bounds on NP could be improved by taking into account different alternatives for decay distributions

Summary

Summary

- LFV provides important test of SM against NP
- Here: Made a model-independent analysis of $\tau \rightarrow \ell\ell\ell$ decays
- Classified different operators in EFT
- Calculated Dalitz distributions for individual structures including interference terms

Conclusion

- Dalitz distributions provide a tool to distinguish NP models complementary to e.g. $\Gamma(\tau \rightarrow 3\ell)/\Gamma(\tau \rightarrow \ell\gamma)$
- $\mathcal{O}(100)$ measured events may be sufficient to distinguish effects from radiative and four-lepton-operators
- Experimental bounds on NP could be improved by taking into account different alternatives for decay distributions

Dimension 8 Operators

Ingredients

- Neglect covariant derivatives: small lepton Yukawa coupling
- Neglect Dim 8 operators due to v^2/Λ^2 suppression

dim=8 leptonic:

$$P_1 = (\bar{L}HR)(\bar{L}HR)$$

$$P_2 = (\bar{L}\tau^a HR)(\bar{L}\tau^a HR)$$

$$Q_1 = (\bar{L}HR)(\bar{R}H^\dagger L)$$

$$Q_2 = (\bar{L}\tau^a HR)(\bar{R}H^\dagger \tau^a L)$$

$$P_1^{(T)} = (\bar{L}H\sigma_{\mu\nu}R)(\bar{L}H\sigma^{\mu\nu}R)$$

$$P_2^{(T)} = (\bar{L}\tau^a H\sigma_{\mu\nu}R)(\bar{L}\tau^a H\sigma^{\mu\nu}R)$$

Radiative Operators

- Photon exchange, where $(s, h) = (L, R)$ respectively (R, L)

$$\frac{e}{4\pi} \frac{v}{\Lambda^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} (\bar{\ell}_h (-i\sigma_{\mu\nu}) \tau_s) F^{\mu\nu}$$

$$\rightarrow \frac{e}{4\pi} \frac{v}{\Lambda^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} (\bar{\ell}_h (-i\sigma_{\mu\nu}) \tau_s) \frac{q^\nu}{q^2} \bar{\ell}' \gamma^\mu \ell''$$

- Z_0 exchange

$$\frac{v}{\Lambda^2} \frac{1}{v^2} (\bar{\ell} \sigma_{\mu\nu} \tau) q^\nu (\bar{\ell}' \gamma^\mu (g_V + g_A \gamma_5) \ell'')$$

Where $(g_A) g_V$: (axial)vector couplings of Z_0 to leptons

\Rightarrow Z_0 exchange suppressed by small Yukawa coupling of tau

$$\frac{Z_{\text{contr.}}}{\gamma_{\text{contr.}}} \propto \frac{q^\nu}{v^2} \frac{q^2}{q^\nu} \propto \frac{m_\tau^2}{v^2}$$

Loop Contributions

- Operators bi-linear in lepton fields, contributing at loop level, e.g. in [\[hep-ph/9510309, hep-ph/0404211, hep-ph/0507001\]](#)
- We neglect possible form factor effects for decays into virtual photons from long-distance lepton or quark loops: In the most general case, the $\tau \rightarrow \ell \gamma^*$ vertex could be parametrized as

$$\frac{e}{4\pi} \frac{v}{\Lambda^2} \sum_{h,s} \bar{\ell}_h \left\{ g_{\text{rad}}^{(s,h)}(q^2) (-i\sigma_{\mu\nu}) q^\mu + m_\tau f_{\text{rad}}^{(s,h)}(q^2) \left(\gamma_\nu - \frac{q_\nu}{q^2} \not{q} \right) \right\} \tau_s$$

where $g_{\text{rad}}^{(s,h)}(0) \equiv g_{\text{rad}}^{(s,h)}$ and $f_{\text{rad}}^{(s,h)}(0) = 0$, see e.g. [\[hep-ph/9710389\]](#)

NP scenarios

- Typical SUSY scenario: photon-dipole operator dominates four-lepton operators [[hep-ph/0404211](#)]:

$$\frac{\Gamma(\tau \rightarrow 3\mu)}{\Gamma(\tau \rightarrow \mu\gamma)} \simeq \frac{\alpha_{\text{em}}}{3\pi} \left(\ln \frac{m_\tau^2}{m_\mu^2} - \frac{11}{4} \right) = \mathcal{O}(10^{-3})$$

- Higgs-mediated $\tau \rightarrow \mu$ in decoupling limit [$\cos(\beta - \alpha) = 0$, $m_{A^0} \gg M_Z$], $\tan\beta$ & off-diagonal slepton mass-matrix element $\delta_{3l} = \tilde{m}_{3l}^2/\tilde{m}^2$ large [[hep-ph/0505046](#)]:

$$\frac{\Gamma(\tau \rightarrow \ell\mu\mu)}{\Gamma(\tau \rightarrow \ell\gamma)} \leq \frac{3 + 5\delta_{\ell\mu}}{36} \sim \mathcal{O}(0.1)$$

- LHT [$M_{\tilde{L}} \sim \mathcal{O}(1 \text{ TeV})$]: Z_0 and box-diagram dominate radiative operators [[hep-ph/0609095](#), [hep-ph/0612327](#), [hep-ph/0702136](#)]:

$$\frac{\Gamma(\tau \rightarrow 3\mu)}{\Gamma(\tau \rightarrow \mu\gamma)} = \mathcal{O}(1)$$

⇒ Sub-dominance of radiative dipole operator: Expect a rather flat Dalitz distribution for $\tau \rightarrow 3\mu$

Minimal Flavour Violation: More Details I

- Here $SU(3)_L \times SU(3)_{ER}$ breaking described by two spurion fields

$$\lambda = \frac{m_\ell}{v} = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau) \quad g_\nu = \frac{\Lambda_{LN}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2}) U^\dagger$$

- $\lambda \sim (\bar{3}, 3)$ describes SM Yukawa interaction
- $g_\nu \sim (\bar{6}, 1)$ stems from dim-5 lepton number violating term

$$\mathcal{L}_{\text{Maj}} = \frac{1}{2\Lambda_{LN}} (N^T g N) \quad \text{where} \quad N = \left(T_3^{(R)} + \frac{1}{2} \right) H^\dagger L$$

- Λ_{LN} large \Rightarrow resulting neutrino masses $m_{\text{Maj}} \sim v^2/\Lambda_{LN}$ are small, even if spurion g_ν has generic entries of order unity
- Interested in 4-lepton processes $L^i L^j L_k^* L_l^*$, $L^i R^j L_k^* R_l^*$, etc.
- Consider minimal spurion insertion to form gauge invariant quantity

Minimal Flavour Violation: More Details II

Constraints for Multiplets

- $\Delta = \Delta^\dagger$ and $G_{ij}^{kl} = G_{ji}^{kl} = G_{ij}^{lk}$, and $\sum_i G_{ij}^{il} = 0$

Tensor Decomposition in Terms of PMNS matrix

- Singlet, no flavour transition: $\text{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\text{LN}}^2}{v^4} (m_{\nu_1}^2 + m_{\nu_2}^2 + m_{\nu_3}^2)$
- Octet: $\Delta = g_\nu^\dagger g_\nu - \frac{1}{3} \text{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\text{LN}}^2}{v^4} U \left(\text{diag}[m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2] - \bar{m}_\nu^2 \right) U^\dagger$
- 27plet:
$$G_{ij}^{kl} = (g_\nu)_{ij} (g_\nu^*)^{kl} - \frac{1}{12} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k \right) \text{tr}(g^\dagger g)$$

$$- \frac{1}{5} \left(\delta_i^a \delta_b^l \delta_j^k + \delta_j^a \delta_b^l \delta_i^k + \delta_i^a \delta_b^k \delta_j^l + \delta_j^a \delta_b^k \delta_i^l \right) \Delta_a^b$$

Minimal Flavour Violation: More Details III

Examples of Spurion Insertions

- 4-Lepton operators $(L^* \Delta L)(L^* L)$ $(L^* \Delta L)(R^* R)$
- Purely right handed operator $(R^* R)(R^* \lambda g^\dagger g \lambda^\dagger R)$
- Radiative operator $(L^* \Delta \lambda^\dagger R)$ and $(R^* \lambda \Delta L)$

Assumptions for PMNS Matrix

- Use PDG Parametrization
- $\sin^2 \theta_{13} \sim \Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2 \ll 1$ and $\theta_{23} = 45^\circ$
- normal neutrino hierarchy ($m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}$)
- inverted hierarchy ($m_{\nu_1} \sim m_{\nu_2} \gg m_{\nu_3}$)