SM Predictions for Muon $(g - 2)/2$

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Outline

1. QED
2. Electroweak term
3. Progress with $e^+e^-$ data
4. LO hadronic term
5. Prospects
6. Errors or New Physics?
7. Conclusions
Muon Anomalous Magnetic Moment

\[ \vec{\mu} = g \frac{e}{2m} \vec{s}, \quad a = (g - 2)/2. \]

In Dirac theory for pointlike particles \( g = 2 \), higher-order effects or new physics \( \Rightarrow g \neq 2 \)

Any significant difference of \( a_\mu^{\text{exp}} \) from \( a_\mu^{\text{th}} \) indicates New Physics beyond the Standard Model.

\( a_\mu \) is much more sensitive to new physics effects than \( a_e \):
the gain is usually \( \sim (m_\mu/m_e)^2 \approx 4.3 \cdot 10^4 \).

\[ a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{NP}}, \quad a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}}. \]
Experimental Status of $a_l$

\[ a_e = 1159652180.73(28) \times 10^{-12} \quad 0.24 \text{ppt} \]

D. Hanneke et al., PRL 100, 120801 (2008)
QED test or $\alpha$ determination

\[ a_\mu = 116592080(63) \times 10^{-11} \quad 0.54 \text{ppb} \]

G.W. Bennett et al. (E821), PRD 73, 072003 (2006)
Sensitive test of the Standard Model

\[ a_\tau = -0.018(17) \quad \text{or} \quad -0.052 < a_\tau < 0.013 \quad 95\% \text{CL} \]

Theory: $117721(5) \times 10^{-8}$, SE, M. Passera, MPL A 22, 159 (2007)
$a_{\mu}^{\text{QED}} \cdot 10^{10} = \sum C_i \left( \frac{\alpha}{\pi} \right)^i = 11614097.3 \ (\text{1-loop}) \quad 1 \ \text{diagram}$

$+ \quad 41321.8 \ (\text{2-loop}) \quad 9$

$+ \quad 3014.2 \ (\text{3-loop}) \quad > 100$

$+ \quad 38.1 \ (\text{4-loop}) \quad > 1000$

$+ \quad 0.4 \ (\text{5-loop}) \quad > 20000$

$\alpha^3$ terms known analytically (S. Laporta, E. Remiddi, 1993),

$\alpha^4$ terms – numerically (T. Kinoshita et al., 2003-2008),

$L \log \alpha^5$ (TK et al., 2005, 2007; A.L. Kataev, 2006, K. Chetyrkin et al., 2008):

$a_{\mu}^{\text{QED}} = (116584719.4 \pm 1.4) \cdot 10^{-11}$.

From the latest value of $a_e$ (D. Hanneke et al., 2008; M. Passera, 2008):

$\alpha^{-1} = 137.035999084(51), \ a_{\mu}^{\text{QED}} = (116584718.09 \pm 0.14 \pm 0.04) \cdot 10^{-11}$.

The errors are due to: a/ $O(\alpha^5)$, b/ $\alpha$
**Electroweak contribution** $a_{\mu}^{\text{EW}}$

One-loop electroweak contributions

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>$a_{\mu}^{\text{EW}}, 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>\ldots, \ldots, \ldots</td>
<td>1972</td>
<td>19.5</td>
</tr>
<tr>
<td>A. Czarnecki et al.</td>
<td>1996</td>
<td>15.2 ± 0.4</td>
</tr>
<tr>
<td>A. Czarnecki et al.</td>
<td>2002</td>
<td>15.4 ± 0.1 ± 0.2</td>
</tr>
</tbody>
</table>

The errors are due to: a/ hadr. loops, b/ $M_H, M_t$, 3-loop effects.
Hadronic contribution $a^\text{had}_\mu$

$$a^\text{had}_\mu = a^\text{had,LO}_\mu + a^\text{had,H0}_\mu + a^\text{had,LBL}_\mu$$

$$a^\text{had,LO}_\mu = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^\infty ds \frac{R(s) \hat{K}(s)}{s^2} ,$$

C. Bouchiat, L. Michel, Bouchiat, 1961; M. Gourdin, E. de Rafael, 1969

$$R(s) = \frac{\sigma(e^+e^-\rightarrow\text{hadrons})}{\sigma(e^+e^-\rightarrow\mu^+\mu^-)} ,$$

$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \rightarrow \infty$,

$1/s^2$ emphasizes the role of low energies,

particularly important is the reaction $e^+e^- \rightarrow \pi^+\pi^-$

with a large cross section below 1 GeV.
Contributions of Various Energy Ranges to $a^\text{had,LO}_\mu$
How is $R(s)$ Measured?

- $\sqrt{s} < 2$ GeV – exclusive modes
  $(\pi^+\pi^-, \pi^+\pi^-\pi^0, \ldots, K\bar{K}, \ldots)$
- Possibly missing (small $\sigma$, undetected) final states
- Above 2 GeV – total $R$
  (all multihadronic events)
- Initial state radiation (ISR), vacuum polarization (VP), final state radiation (FSR):
  M. Drees, K. Hikasa, 1990

\[ \sigma_{\text{dr}} = \frac{N}{\int L dt \cdot \varepsilon (1+\delta_{\text{ISR}})} \]
\[ \sigma_{\text{bare}} = \sigma_{\text{dr}} |1 - \Pi(s)|^2 (1 + \delta_{\text{FSR}}) \]
$e^+e^- \rightarrow \pi^+\pi^-$ (CMD-2, SND and KLOE)

CMD-2: $\sim 9 \cdot 10^5$ ev.

<table>
<thead>
<tr>
<th>2E, MeV</th>
<th>$\sigma$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>370-520</td>
<td>0.7</td>
</tr>
<tr>
<td>600-970</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>1040-1380</td>
<td>1.3-4.2</td>
</tr>
</tbody>
</table>

SND: $\sim 8 \cdot 10^5$ ev.

<table>
<thead>
<tr>
<th>2E, MeV</th>
<th>$\sigma$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>390-420</td>
<td>3.2</td>
</tr>
<tr>
<td>430-970</td>
<td>1.3</td>
</tr>
</tbody>
</table>

KLOE: $\sim 3.1 \cdot 10^6$ ev.

(590-970) MeV - 0.9%

Expecting news from KLOE (G.Venanzoni) and BaBar (M.Davier)
Most of these channels have also been studied at SND with consistent results.
Good agreement with SND. BaBAR’s points much higher than at DM2

<table>
<thead>
<tr>
<th>Source</th>
<th>Before BaBAR</th>
<th>All + BaBAR</th>
<th>All-DM2+BaBAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta a_\mu$, $10^{-10}$</td>
<td>$2.45 \pm 0.26 \pm 0.03$</td>
<td>$2.79 \pm 0.19 \pm 0.01$</td>
<td>$3.25 \pm 0.09 \pm 0.01$</td>
</tr>
</tbody>
</table>
Good agreement with CMD-2/SND.
Error of continuum below 2 GeV improved.
\(e^+e^- \rightarrow 6\pi\)

\[\delta a_\mu \text{ changed from } 0.10 \pm 0.10 \text{ to } 0.108 \pm 0.016 \text{ and from } 1.42 \pm 0.30 \text{ to } 0.890 \pm 0.093 \]

\text{Significant improvement compared to the previous data!}
Excellent agreement with the pQCD predictions!
**$e^+e^-$ Data Based Calculation of $a_{μ}^{\text{had},\text{LO}}$ (DEHZ-2006)**

<table>
<thead>
<tr>
<th>$\sqrt{s}$, GeV</th>
<th>$a_{μ}^{\text{had},\text{LO}}, 10^{-10}$</th>
<th>$\delta a_{μ}^{\text{had},\text{LO}}, %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2π</td>
<td>504.6 ± 3.1 ± 1.0</td>
<td>73.0</td>
</tr>
<tr>
<td>$\omega$</td>
<td>38.0 ± 1.0 ± 0.3</td>
<td>5.5</td>
</tr>
<tr>
<td>$\phi$</td>
<td>35.7 ± 0.8 ± 0.2</td>
<td>5.2</td>
</tr>
<tr>
<td>0.6 – 1.8</td>
<td>54.2 ± 1.9 ± 0.4</td>
<td>7.8</td>
</tr>
<tr>
<td>1.8 – 5.0</td>
<td>41.1 ± 0.6 ± 0.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$J/\psi, \psi'$</td>
<td>7.4 ± 0.4 ± 0.0</td>
<td>1.1</td>
</tr>
<tr>
<td>$&gt; 5.0$</td>
<td>9.9 ± 0.2 ± 0.0</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>690.9 ± 3.9$<em>{\exp}$ ± 1.9$</em>{\text{rad}}$ ± 0.7$_{\text{QCD}}$</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Higher accuracy of $e^+e^-$ data: the $a_{μ}^{\text{had},\text{LO}}$ error is 4.4 (0.63%) compared to 15.3 of EJ, 1995 and 7.2 of DEHZ, 2003!
Various $a_{\mu}^{\text{had,LO}}$ Calculations

<table>
<thead>
<tr>
<th>Calculation</th>
<th>$a_{\mu}^{\text{had,LO}}, 10^{-10}$</th>
<th>$\sqrt{s_{\text{PQCD}}}, \text{GeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EJ, 1995</td>
<td>$702.3 \pm 15.3$</td>
<td>40</td>
</tr>
<tr>
<td>DEHZ, 2003</td>
<td>$696.3 \pm 7.2$</td>
<td>5</td>
</tr>
<tr>
<td>DEHZ, 2006</td>
<td>$690.9 \pm 4.4$</td>
<td>5</td>
</tr>
<tr>
<td>HMNT, 2008</td>
<td>$689.4 \pm 4.6$</td>
<td>11.09</td>
</tr>
<tr>
<td>J, 2008</td>
<td>$692.3 \pm 6.0$</td>
<td>12</td>
</tr>
</tbody>
</table>
The contributions of all 3 graphs can be calculated in terms of the
\( \int R(s)G(s)ds/s^{2(3)} \), where \( G(s) \) is a smooth function of \( s \), so that the low energy range again dominates the integral. Several calculations agree. The accepted value is (B. Krause, 1997; K. Hagiwara et al., 2003):

\[
a_{\mu}^{\text{had,HO}} = (-9.8 \pm 0.1) \cdot 10^{-10}.
\]
Light-by-Light Scattering

Various approaches used:

- Vector Dominance and Chiral models
- Data on $\gamma\gamma^* \to \pi^0, \eta, \eta'$ (single-tag)
- Effective field theory

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>$a^{1b1}_\mu, 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J. Bijnens et al.</td>
<td>1996 (2002)</td>
<td>8.3 ± 3.2</td>
</tr>
<tr>
<td>K. Melnikov and A. Vainshtein</td>
<td>2003</td>
<td>13.6 ± 2.5</td>
</tr>
<tr>
<td>M. Davier and W. Marciano</td>
<td>2004</td>
<td>12.0 ± 3.5</td>
</tr>
<tr>
<td>J. Bijnens and J. Prades</td>
<td>2006</td>
<td>11.0 ± 4.0</td>
</tr>
<tr>
<td>F. Jegerlehner and A. Nyffeler</td>
<td>2008</td>
<td>11.9 ± 4.0</td>
</tr>
</tbody>
</table>
### Theory vs Experiment – I

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$a_\mu, \ 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>11659208.0 ± 6.3</td>
</tr>
<tr>
<td>QED</td>
<td>11658471.8 ± 0.016</td>
</tr>
<tr>
<td>Electroweak</td>
<td>15.4 ± 0.1 ± 0.2</td>
</tr>
<tr>
<td>Hadronic</td>
<td>693.1 ± 5.6</td>
</tr>
<tr>
<td>Theory</td>
<td>11659180.3 ± 5.6</td>
</tr>
<tr>
<td>Exp.–Theory</td>
<td>27.7 ± 8.4 (3.3σ)</td>
</tr>
</tbody>
</table>

The difference between experiment and theory is 3.3σ!
### Theory vs Experiment – II

<table>
<thead>
<tr>
<th>Calculation</th>
<th>$a_{\mu}^{\exp}, 10^{-10}$</th>
<th>$a_{\mu}^{\text{th}}, 10^{-10}$</th>
<th>$a_{\mu}^{\exp} - a_{\mu}^{\text{th}}, 10^{-10} \ (\sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EJ, 1995</td>
<td>11659230 ± 84</td>
<td>11659210 ± 16</td>
<td>20 ± 86 (---)</td>
</tr>
<tr>
<td>DEHZ, 2003</td>
<td>11659203 ± 8</td>
<td>11659180.9 ± 8.0</td>
<td>22.1 ± 11.3 (1.9)</td>
</tr>
<tr>
<td>DEHZ, 2006</td>
<td>11659208.0 ± 6.3</td>
<td>11659180.3 ± 5.6</td>
<td>27.7 ± 8.4 (3.3)</td>
</tr>
<tr>
<td>HMNT, 2008</td>
<td>11659208.0 ± 6.3</td>
<td>11659180.4 ± 5.1</td>
<td>27.6 ± 8.1 (3.4)</td>
</tr>
<tr>
<td>J, 2008</td>
<td>11659208.0 ± 6.3</td>
<td>11659181.3 ± 7.2</td>
<td>26.7 ± 9.6 (2.8)</td>
</tr>
</tbody>
</table>
Theory vs Experiment – III
How Real Is $\alpha_{\mu}^{\text{had}}$ Accuracy?

- Missing states: neutrals; $\pi^+\pi^- n\pi^0$, $K\bar{K} n\pi$ - isospin
- New states from BaBar, double counting
- Radiative corrections (FSR):
  Charge asymmetry at KLOE
  $3k e^+e^- \rightarrow \pi^+\pi^-\gamma$ evts at CMD-2
- Correlations
- Averaging
- Light-by-light term
- Double counting (LO and HO)

Charge asymmetry in $e^+e^- \rightarrow \pi^+\pi^-\gamma$

sQED is OK ($0.6 < M_{\pi\pi}^2 < 0.7$ GeV$^2$)
CVC. $e^+e^- \rightarrow X^0$ and $\tau^- \rightarrow \nu_\tau X^-$

Allowed $I^G J^P = 1^+ 1^-$:

$X^- = \pi^- \pi^0, (4\pi)^-, \omega \pi^-, \eta \pi^- \pi^0, K^- K^0, (6\pi)^-, \ldots$

Large SU(2) breaking corrections from theory, V. Cirigliano et al., 2002

$M(\Gamma)_\rho^0 \neq M(\Gamma)_\rho^\pm$ helps,

M. Davier, 2003; S. Ghozzi, F. Jegerlehner, 2004
Corrections to the $\tau$ Spectral Functions

- $S_{EW} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $m_{\pi^\pm} \neq m_{\pi^0}$
  (phase space, $\Gamma_\rho$)
- $m_{\rho^\pm} \neq m_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays
  ($\pi\pi\gamma$, $\pi(\eta)\gamma$, $l^+l^-$)
- $m_u \neq m_d$
  and 2 class currents

V. Cirigliano, G. Ecker, H. Neufeld, 2002
M. Davier, S. Eidelman, A. Höcker, Z. Zhang, 2002

G. Lopez Castro, 2006
The branching from all groups is systematically higher than the CVC prediction: $\mathcal{B}_\tau - \mathcal{B}_{ee} = (0.94 \pm 0.21)\%$.
The discrepancy – a 3.6% effect, twice the SU(2) correction.

The puzzle remains unsolved $\Rightarrow \tau$ data not used

M. Benayoun, EPJ C55, 199 (2008):
more complete SU(2) breaking, mixing, no conflict
New data on $\tau^- \to \pi^-\pi^0\nu_\tau$ from Belle

$B_{\text{Belle}} = (25.24 \pm 0.04 \pm 0.39)\%$  
$B_{\text{ALEPH}} = (25.471 \pm 0.097 \pm 0.085)\%$

The contributions to $a^\text{had}_\mu$ are also compatible due to compensation at tails
What can be done from the $e^+e^-$ side?

More ISR analysis from KLOE, BaBar, Belle; better $R$ below 4.3 GeV from CLEO-c: $4.4 \rightarrow 2.8$

Experiments at VEPP-2000 with 2 detectors up to $\sqrt{s}=2$ GeV with $L_{\text{max}} = 10^{32} \text{ cm}^{-2}\text{s}^{-1}$, $10^{30} \text{ cm}^{-2}\text{s}^{-1}$ achieved this June

A similar machine (DAΦNE-II) is discussed in Frascati, a $\tau - c$ factory in Beijing commissioning.

By 2012: $2.8 \rightarrow 2.2$, the total error of 4.6 limited by the LBL term (4.0)
Future of \((g_\mu - 2)/2 - II\)

1. New \((g_\mu - 2)/2\) experiment at FLAB expects the accuracy of \(0.1\text{ppm}_{\text{stat}}\) and \(0.1\text{ppm}_{\text{syst}}\) or \(1.5 \times 10^{-10}\)

2. Such accuracy for \(a_{\text{had},\text{LO}}\) corresponds to 0.2\%, hardly ever achievable for the absolute measurement of \(\sigma(e^+e^- \rightarrow \text{hadrons})\)

3. \(a_{\mu}^{\text{had}}\) calculation from 1st principles (QCD, Lattice).
   QCD instanton model (A. Dorokhov, 2003),
   Recently from the Lattice: \(a_{\mu}^{\text{had}} = (545 \pm 65) \cdot 10^{-10} \Rightarrow (667 \pm 20) \cdot 10^{-10}\)
   (C. Aubin and T. Blum, 2005), attempts to estimate \(a_{\mu}^{\text{lbl}}\) (M. Hayakawa et al., 2005).
Explaining the Discrepancy: Errors or New Physics?

M. Passera, W. J. Marciano, A. Sirlin, 2008

- The lbl term is wrong ⇒ Should move it by 8-10σ
- If assume errors in \( \sigma(s) \) and increase it, \( \Delta \alpha_{\text{had}}^{(5)}(M_Z) \) also increases ⇒ \( M_H^{UB} \) decreases restricting a narrow allowed region \( 114 \text{ GeV} < M_H < 154 \text{ GeV} \)
- Using \( \tau \) data also increases \( \Delta \alpha_{\text{had}}^{(5)}(M_Z) \) and leads to \( M_H < 133 \text{ GeV} \)
- To bridge \( \Delta a_\mu \), \( \sigma(s) \) should be increased by 4% from threshold to infinity. Then \( M_H < 70 \text{ GeV} \). If \( \sigma(s) \) is increased locally, \( M_H < 130 \text{ GeV} \).
- All scenarios look rather unlikely ⇒ New Physics?
Conclusions

- BNL success stimulated significant progress of experiment and theory
- QED and EW terms are in good shape
- Improved $e^+e^-$ data ⇒ smaller $\delta a_{\mu}^{\text{had},\text{LO}}$ matching experiment
- $\tau$ data could further improve the accuracy but a serious yet unexplained failure of CVC relations for $e^+e^-$ and $\tau$ is observed
- Further improvement in $a_{\mu}^{\text{had},\text{LO}}$ by a factor of 2 will be possible after VEPP-2000, DAΦNE-II, CESRc and $(c-\tau)$ factory + ISR at DAΦNE and B-factories
- Light-by-light term will soon limit the accuracy
- More theory input needed
- $a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}}$ differs from 0 by $\sim 3\sigma$ or more ⇒ A hint to New Physics?
### Calculations of $a_{\mu}^{\text{had,LO}}$

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>$a_{\mu}^{\text{had,LO}}, 10^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.Bouchiat, L.Michel</td>
<td>1961</td>
<td>$\simeq 648$</td>
</tr>
<tr>
<td>M.Gourdin, E. de Rafael</td>
<td>1969</td>
<td>$650 \pm 50$</td>
</tr>
<tr>
<td>A.Bramon et al.</td>
<td>1972</td>
<td>$680 \pm 90$</td>
</tr>
<tr>
<td>J.Calmet et al.</td>
<td>1977</td>
<td>$702 \pm 80$</td>
</tr>
<tr>
<td>T.Kinoshita et al.</td>
<td>1985</td>
<td>$707 \pm 18$</td>
</tr>
<tr>
<td>S.Eidelman, F.Jegerlehner</td>
<td>1995</td>
<td>$702 \pm 15$</td>
</tr>
<tr>
<td>R.Alemany et al.</td>
<td>1998</td>
<td>$701.1 \pm 9.4$</td>
</tr>
<tr>
<td>M.Davier, A.Höcker</td>
<td>1998</td>
<td>$692.4 \pm 6.2$</td>
</tr>
</tbody>
</table>
Light-by-Light Scattering – II

- A. Pivovarov, 2001: $\sim 14.3 \cdot 10^{-10}$
  $u, d, s$ with $m_q$, heavy quarks with standard masses

- J.F. de Trocóniz, F.J. Ynduráin, 2001: $(9.2 \pm 2.0) \cdot 10^{-10}$
  Constituent quarks + $\pi^0$

- A. Dorokhov, 2005: $(10.6 \pm 1.0) \cdot 10^{-10}$
  Instanton liquid model

- J. Erler, G. Toledo Sánchez, 2006: $< 15.9 \cdot 10^{-10}$
  Parton model

Is there a way to consistently relate experimental data on
$\gamma\gamma^* \rightarrow X^0$ with $J^{PC} = 0^{-+}, 0^{++}, 1^{++}, 2^{--}, 2^{++}$ to the corresponding contributions to $a_{\mu}^{\text{had,LBL}}$?
### Branchings of $\tau^- \rightarrow X^- \nu_\tau$ Decay, %

<table>
<thead>
<tr>
<th>Hadronic State $X$</th>
<th>Experiment, 2002</th>
<th>CVC Prediction</th>
<th>$B_{\text{exp}} - B_{\text{CVC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^- \pi^0$</td>
<td>25.31 ± 0.18</td>
<td>24.76 ± 0.25</td>
<td>0.55 ± 0.31</td>
</tr>
<tr>
<td>$\pi^- 3\pi^0$</td>
<td>1.08 ± 0.10</td>
<td>1.07 ± 0.05</td>
<td>0.01 ± 0.11</td>
</tr>
<tr>
<td>$2\pi^- \pi^+ \pi^0$</td>
<td>4.19 ± 0.23</td>
<td>3.84 ± 0.17</td>
<td>0.35 ± 0.29</td>
</tr>
<tr>
<td>$\omega \pi^-$</td>
<td>1.94 ± 0.07</td>
<td>1.82 ± 0.07</td>
<td>0.12 ± 0.10</td>
</tr>
<tr>
<td>Total</td>
<td>31.59 ± 0.31</td>
<td>30.28 ± 0.34</td>
<td>1.31 ± 0.46</td>
</tr>
</tbody>
</table>

With more accurate data some deviations have been observed.
Corrections to the $\tau$ Spectral Functions

- $S_{EW} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $m_{\pi^\pm} \neq m_{\pi^0}$
  (phase space, $\Gamma_\rho$)
- $m_{\rho^\pm} \neq m_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays
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- $m_u \neq m_d$
  and 2 class currents

V. Cirigliano, G. Ecker, H. Neufeld, 2002
M. Davier, S. Eidelman, A. Höcker, Z. Zhang, 2002
The branching from all groups is systematically higher than the CVC prediction: $B_\tau - B_{ee} = (0.94 \pm 0.32)\%$!
$2\pi$ Spectral Functions in $e^+e^-$ and $\tau$ after KLOE and SND

The puzzle remains unsolved $\Rightarrow$ $\tau$ data not used
CVC in the $4\pi$ Channel. $e^+e^-$ vs. $\tau$

$2\pi^+2\pi^-$

$\pi^+\pi^-2\pi^0$

$\mathcal{B}(\tau), \% \quad 1.01 \pm 0.08$

$\mathcal{B}(\text{CVC}), \% \quad 1.09 \pm 0.08$

$\Delta \mathcal{B}, \% \quad -0.08 \pm 0.11$

$4.54 \pm 0.13$

$3.63 \pm 0.21$

$+0.91 \pm 0.25$
Why are $e^+e^-$ and $\tau$ Spectral Functions Different?

- Problems with data: underestimated systematics, normalization, rad. corr.
- Problems with SU(2) breaking corrections; Is ChPT reliable? The uncertainty of corrections may be large (K.Maltman, 2005)
- Non (V-A) contribution to e/w interactions (M.Chizhov, 2003) inspired by problems in $\pi^+ \rightarrow e^+\nu_e\gamma$ (E.Frlez et al., 2003)
- Effect of charged Higgs propagator in $\tau$ decay
- $m_{\rho^\pm} > m_{\rho^0}$ by a few MeV (S.Ghozzi and F.Jegerlehner, 2003, M.Davier, 2003). Current experiments indicate equality within a few MeV.
The design luminosity $\mathcal{L} = 10^{32} \text{ cm}^{-2}\text{s}^{-1}$, with $\int L dt \approx 1 - 2 \text{ fb}^{-1}$ during 3–5 years $\Delta a_{\mu}^{\text{had}}/a_{\mu}^{\text{had}}$ can be improved by a factor of 2!